ANSWERS TO THE EXERCISES,

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THE SCIENCE AND ART

OF

ARITHMETIC;

For the Use of Schools.

PART I. INTEGRAL.

BY

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AND

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"The mills of God grind slowly, but they grind exceeding small."

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TO

MISS PIPE,

OF LALEHAM, CLAPHAM PARK,

This Work is inscribed,

IN TOKEN OF THEIR RESPECT AND GRATITUDE,

BY

THE AUTHORS.

PREFACE.

THERE is no need now to insist on a rational study of Arithmetic. It is admitted on all sides that no subject is so well fitted for the early training of the reasoning powers, and principally because the student is enabled, without apparatus of any kind, steadily to test all his a priori conclusions by the light of experience. In History, Physics, and even in Language, the student must have premisses supplied him; but his Mathematical studies can all be "evolved from his inner consciousness."

Ever since the pernicious plan of teaching by mere rote and rule of thumb was abandoned, the teaching of Elementary Mathematics has steadily risen towards higher levels, and we may perhaps be allowed to note down some of the most remarkable stages. A certain school of teachers very early felt the necessity of enlisting the child's reason on their side; but the means they adopted were not always wise or even honest. In a modest little work on Vulgar Fractions, which is otherwise very meritorious, we find the following "proof" of the formula $\frac{a}{b}=\frac{ma}{mb}$:—$\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{5}{5}=\frac{6}{6}=\frac{7}{7}=\frac{8}{8}$, &c. If of $\frac{2}{2}$ both terms be multiplied by 3, we obtain $\frac{6}{6}$; by 4, $\frac{8}{8}$; and so on. If, on the contrary, of $\frac{10}{10}$, both terms be divided by 5, we obtain $\frac{2}{2}$, &c.; hence (!), &c. &c.

We can imagine that inexperienced children would readily give their assent to such a proof; but the teacher ought to have known, 1st, that $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, &c., are only integers very thinly disguised as fractions, and that it does not follow that what is true of integers is

necessarily true of fractions; 2nd, that from this proof we might equally deduce $\frac{a}{b} = \frac{a \pm c}{b \pm c}$, which, though true for these disguised units, is false for fractions.

Another school of teachers have resolutely shrunk from such slurring over of difficulties, and are satisfied with nothing short of irrefragable proof. This is, of course, a great step in advance, but they fail in one point. The proof in each case is *given* to the student, and he has to learn it. Now following and even mastering and remembering a chain of reasoning, though it certainly is an instruction and even a discipline, is not yet true education.

The disciples of Professor De Morgan well know that "each new notion to be acquired must be attached to and assimilated with the notions already existing in the mind." The logical consequence of this maxim is, that the student must be led to the *discovery* of the several rules by some path such as an original discoverer might have travelled. Thus each rule furnishes the *raison d'être* of its successor; for example, Subtraction must lead through Cumulative Subtraction to Division, which, in its turn, leads to Fractions, and these again lead to the idea of Ratio and Proportion. How absurd, then, were those systems which taught Proportion before Fractions; the advanced notion before the earlier one! The steady educing of new and wider notions from old and narrow ones is true education.

Some might object to this method as being slow in shewing results. Even though we must admit the necessity of "payment by results;" yet we hold that for educational purposes the "seeking of the truth is worth more than the finding of it." In other words, Processes are worth more than Results; the highest wages of work is work itself.

But we deny that the method of teaching here advocated is slow and barren of results. Progress is not synonymous with mere advance from rule to rule. A thorough and all but visual realization of each process, gives the young student a feeling of power and perfect mastery, which obviates the necessity of perpetual recapitu-

lation. An experience of upwards of twenty years has shewn us that the more nearly we have at any time approximated to this our ideal, the better the results have been, even when subjected to the severe test of a competitive examination.

In the following pages we have endeavoured to carry out these principles. The attempt is doubtless imperfect enough; but all shortcomings we trust will be treated indulgently, if we have made but one step further in the right direction. The book is intended to guide young teachers; for although the pupil must use the work for the sake of the examples, he is pretty sure not to read the letter-press. We hope, however, that the teacher will find his work, if not lessened, at least rendered easier by it. The sequence of Chapters is not necessarily a sequence of lessons; indeed, we should recommend that two or more Chapters be taken in hand simultaneously.

The First Part treats of Integral Arithmetic purely; the Second Part, of Vulgar Fractions and the notions immediately deduced from them. The Third Part is devoted to Approximate Calculations, and new methods are given for very rapid, and in most cases mental, decimalization of money, weights and measures, to any assigned degree of accuracy, and for the ready interconversion of the coins, weights and measures of different nations. We have reason to believe that these methods will prove a considerable boon to the commercial world.

A. SONNENSCHEIN,
H. ARTHUR NESBITT.

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ARITHMETIC.

CHAPTER I.

NUMERATION AND NOTATION.

1. The word Arithmetic is derived from the Greek word *Arithmos*, number, and is therefore the study of numbers, which divides itself naturally into two branches, viz. such calculations as can be done without the aid of signs or symbols, and such as require this aid. The former is called Mental Arithmetic; the latter, Slate Arithmetic, or, more properly, Written or Symbolic Arithmetic. It is of this latter branch that we propose chiefly to treat.

2. The word symbol also is Greek, and has the same force as the Latin "sign" or the Germanic "mark;" the figures, 1, 2, 3, &c., being symbols, signs or marks of their respective quantities. Symbols have a language of their own, amenable to all the rules of logic, to the expression of which rules they are especially adapted; and we are endeavouring, in the study of Arithmetic, to acquire a first knowledge of this peculiar language.

3. When we reckon with counters, say pebbles, each pebble may be made to represent one or more things: if we had, for example, to sum up the number of oxen in two herds, we might count two collections of pebbles, each of which should be equal in number to one of the herds of oxen. There would be no connection between a pebble and an ox further than this, that the pebble is a representative or a *symbol* for the ox. Similarly, written strokes or dots may be made to render a like service, and hence our whole system of Symbolic Arithmetic. [The word *calculation* is derived from the

Latin *calculus*, a pebble, and the professional calculator in ancient Rome, by means of slave labour, adopted some such mechanical method of computation.]

4. The first step necessary to an advance in this study was to agree upon a fixed set of symbols, which should be the alphabet of the common language. Several such attempts have been made; the two sets of symbols in present use are the Roman and the Arabic. The principal Roman symbols are I, V, X, L, C, D, M, signifying respectively one, five, ten, fifty, one hundred, five hundred, and one thousand.*

EXERCISE A.

Read from the board the following:

XII, XXV, XXXVIII, L, LX, LXV, LXXIII, LXXXVII, C, CC, CCL, CCCLX, CCCCIII, D, DL, DCLXXXVI, M, MC, MD, MDC, MDCLXXXVIII, MDCCLX, MDCCCXV, MDCCCLXVII.

5. The Romans made a remarkable addition to their scheme of notation, by making the function of the symbol occasionally depend on its position: thus, VI denotes five *and* one, or six; but IV denotes five *diminished by* one, or four. If, then, the smaller quantity precede the larger, it is to be taken away from the larger; if it follow, to be added; consequently, XL denotes forty, and LX sixty.

EXERCISE B.

Read from the board:

XL, LX, XC, CX, MDXCIV, MDCCLXXXIX, MDCCCXLVIII, MDCCCLXIV.

6. These symbols are of little use for complicated calculations, for many reasons, but chiefly for this; that their value depends on their shape, and not at all on their position. The set of symbols in common use relies for its power of expressing any number, however large, almost exclusively on the artifice that the value of each figure increases tenfold for every place it is moved further to the left, and this has entailed the necessity of introducing a symbol for *nothing*,

* For an account of the origin of these and other numerical symbols, see Penny Cyclop., art. Numeral Characters.

viz. 0,* and so important is this symbol, that from the word cipher, the whole art has derived its name, Ciphering. These symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and they are called Arabic Figures, because they came to us from the Moors of Spain, the most cultivated people of the Middle Ages. They, however, were not the inventors, having received them from the Hindoos,† among whom they were in use about the commencement of the Christian era.

7. We may write down any number with only two symbols, viz. 0 and 1, if we agree that the value of the symbol 1 shall be doubled every time we move it one step further to the left. Thus,

one	=	1,		
two	=	10,		
three	=	11,	being	one two and one.
four	=	100,	„	a double two.
five	=	101,	„	a double two and one.
six	=	110,	„	a double two and a two.
seven	=	111,	„	a double two, a two, and one.
eight	=	1000,	„	the four doubled, &c.

EXERCISE C.

Write down all the numbers up to 65 on this system of notation, which is called the Binary Scale, from the Latin *bini*, two at a time.

8. Similarly we may write down any number with three symbols, viz. 0, 1, 2, if we agree that the value of each symbol shall be trebled every time we move it one step further to the left. Thus,—

one	=	1	six	=	20
two	=	2	seven	=	21
three	=	10	eight	=	22
four	=	11	nine	=	100
five	=	12	&c.		

* Read *naught*, which means *nothing*, and not *aught*, which means *anything* (cf. "For aught I know").

† "Dutch" clocks only *come* from Holland, but are *made* in the Black Forest; similarly "Hamburg" grapes *come* from Hamburg, but do not *grow* there.

9. Or we may employ the four symbols 0, 1, 2, 3, if we agree that the value of each symbol shall be quadrupled every time we move it one step further to the left.

10. The Scale of Notation requiring three symbols (0, 1, 2) is called the Ternary Scale; that with four symbols (0, 1, 2, 3) the Quaternary Scale. Similarly we have the Quinary, Senary, Septenary, Octonary, Nonary, Denary or Decimal, Undecimal, Duodecimal Scales, &c., requiring respectively 5, 6, 7, 8, 9, 10, 11, and 12 symbols. The Undecimal and Duodecimal Scales would of course require new symbols, viz. for ten in the Undecimal, and for ten and eleven in the Duodecimal Scale. The most useful of all these is the Decimal Scale, because, owing to our having ten fingers, we count by tens; thus eleven means one and ten; twelve means two and ten; thirteen means three and ten; fourteen, four and ten, &c.; twenty means twain tens; twenty-one, two tens and one, &c.

Exercise D.

On the Ternary Scale write all the numbers up to 82; on the Quaternary, up to 65; on the Quinary, up to 126; on the Senary, to 37; on the Duodecimal (making *t* stand for ten and *e* for eleven), up to 145.

Note. **The teacher will find it advisable to deal with concrete rather than with abstract numbers; and for this reason it is well, in teaching the Decimal Scale of Notation, to lay before the students some illustration, such as a French Metre, (39·37 inches), divided into decimetres, centimetres, and millimetres, the last being taken as the unit. By this means a Metre supplies a thousand units, and a Kilometre (a distance of rather more than half a mile) a million units. This will give an idea, not only of relative, but of absolute magnitude. The authors have derived advantage in teaching very young students from a set of ten large bags, each containing ten little ones, of which each again contains ten papers with ten barleycorns in each paper.**

11. On Decimal Notation.

(*a*) Write down all the numbers up to one hundred.
(*b*) Read off 43.

Question. Why does this represent 43 and not 7?

*Answer.** Because in the Decimal Scale the value of the symbol is increased tenfold by moving the symbol one place to the left, so the 4 represents four tens, the 3 three units; [or on the metre, 43 stands for four centimetres and three millimetres.]

Q. If you had to arrange 43 marbles in heaps of ten each, how many such heaps would there be?

A. Four heaps, and three marbles over.

Q. Would not the three marbles give another heap?

A. No; I am seven marbles short.

Q. If these seven marbles were added, how many heaps of ten would there then be?

A. Five heaps.

Q. Write that down.

A. 50. (And so on with other numbers under one hundred.)

Q. Read off 99.

A. Ninety-nine.

Q. Why ninety-nine, and not eighteen?

A. Because it means nine tens and nine ones.

Q. If you had to draw ninety-nine pounds from the Bank, how many ten-pound notes should you receive?

A. Nine ten-pound notes.

Q. And nothing more?

A. Yes, and nine sovereigns in gold.

Q. Would not these make up another ten-pound note?

A. No; I should be one pound short.

Q. If that pound were supplied, how many ten-pound notes would you then have?

A. Ten ten-pound notes.†

Q. Write that down.

A. £100.

* These answers will not be obtained fully at first. The questions must be so amplified as to lead to them, the requisite amount of amplification depending on the ability of the class; but the subject should not be carried further until each step is thoroughly mastered.

† Never allow the answer "ten," but insist on a reply to the question, "ten what?" *Ans.* Ten pounds, ten times, &c., as the case may be.

Q. How many tens are there in one hundred?

A. Ten tens.

Q. How many units are there in one hundred?

A. One hundred units.

Q. Read off 173.

A. One hundred and seventy-three.

Q. Why should this mean one hundred and seventy-three, and not eleven?

A. Because it means one hundred, seven tens, and three ones.

Q. If 173 marbles were arranged in heaps of ten marbles each, how many such heaps would there be?

A. Seventeen heaps and three marbles over.

Q. Prove it.

A. Because one hundred gives ten tens, and seven more tens are seventeen tens.

[*Q.* Represent this quantity on the metre.

A. One decimetre, seven centimetres, and three millimetres.]

Q. Would not these three units give another ten?

A. No; there are seven units short.

Q. If these seven units were added, how many tens would you then have?

A. Eighteen tens.

Q. How many hundreds can be obtained from 173 units?

A. One hundred, and seventy-three units over.

Q. Would not the 73 units yield another hundred.

A. No; I am twenty-seven units short.

Q. If these 27 units were supplied, how many hundreds would you then have?

A. Two hundreds. (And so on with other numbers under one thousand.)

Q. Read off 999.

A. Nine hundred and ninety-nine.

Q. Why should this mean nine hundred and ninety-nine, and not twenty-seven?

A. Because it represents nine hundreds, nine tens and nine units.

Q. If 999 marbles were arranged in heaps of ten, how many such heaps would there be?

A. Ninety-nine heaps of ten, and nine marbles over.

Q. Prove it.

A. Nine hundreds yield ninety tens, and nine more tens make ninety-nine tens.

Q. Would not the nine units which are over give another ten?

A. No; I am one unit short.

Q. If that unit were supplied, how many tens would you then have?

A. One hundred tens.

Q. How many hundreds could you get out of 999 marbles?

A. Nine hundreds, and ninety-nine marbles over.

Q. Would not these ninety-nine marbles yield you another hundred?

A. No; I am one marble short.

Q. If that marble were supplied, how many hundreds would you then have?

A. Ten hundreds.

Q. Write it down.

A. 1000.

Q. Read off 5743. (And so on with numbers under 10000, as was done with the numbers 43 and 173.)

After some practice the pupils can be trained to answer as follows:

Q. Read off 5872.

A. Five thousand, eight hundred and seventy-two units; or five hundred and eighty-seven tens, and two units over, wanting eight units to make up 588 tens; or fifty-eight hundreds, and seventy-two units over, wanting 28 units to make up fifty-nine hundreds; or five thousands, and eight hundred and seventy-two units over, wanting one hundred and twenty-eight units to make up six thousands.

Next, treat 9999 as 99 and 999 have been treated, and work as before till all numbers under 100000 are rendered familiar. Next, treat 99999 and numbers up to 1000000 similarly. Beyond this it will not be found necessary to go, as it is difficult to form a distinct notion even of this number.

12. The following table of symbolic headings to the several columns should be learnt by heart gradually, as the necessity arises, during the progress of the study of section 11.

$$
\begin{array}{llllll}
X & = X.I & & & & \\
C & = X.X & = C.I & & & \\
M & = X.C & = C.X & = M.I & & \\
XM & = X.M & = C.C & = M.X & = XM.I & \\
CM & = X.XM & = C.M & = M.C & = XM.X & = CM.I \\
MM \text{ or } M^2 & = X.CM & = C.XM & = M.M & = XM.C & = CM.X = M^2.I
\end{array}
$$

NOTE. This table is read:

Ten equals ten times one.

One hundred equals ten times ten, equals a hundred times one.

One thousand equals ten times a hundred, or a hundred times ten, or a thousand times one.

One ten-thousand (a myriad) equals ten times a thousand, or a hundred times a hundred, or a thousand times ten, or ten thousand times one.

One hundred-thousand (a lac) equals ten times a ten-thousand, or a hundred times a thousand, or a thousand times a hundred, or ten thousand times ten, or a hundred thousand times one.

One million equals ten times a hundred-thousand, or a hundred times a ten-thousand, or *a thousand times a thousand*, or ten thousand times a hundred, or a hundred thousand times ten, or a million times one.

[Point out that just as a *balloon* is a big ball, a saloon a big *salle*, so a million is a big mille, viz. a thousand thousand, and that this nomenclature is analogous to the word gross, a big dozen, viz. a dozen dozen.]

13. Hitherto we have treated Numeration analytically; we proceed to the synthesis.

Learn by heart: *Ten is a one-cipher number occupying the second place.*

Write down forty, seventy, ninety, sixty-eight, fifteen, thirty-nine, &c.

A hundred is a two-cipher number occupying the third place.

Write down one hundred, five hundred, seven hundred, eight hundred and forty-three, seven hundred and fifty-nine, six hundred and twenty, six hundred and two, two hundred and six, two hundred and sixteen, six hundred and twelve, six hundred and twenty-one, &c.

A thousand is a three-cipher number occupying the fourth place, a

ten thousand a four-cipher number occupying the fifth place, a hundred thousand a five-cipher number occupying the sixth place, and a million a six-cipher number occupying the seventh place. [Rest at each stage and give examples on it, as has been done in the first two, until that stage is perfectly mastered.]

14. Unity followed by any number of ciphers exceeds by unity the quantity expressed by the same number of nines: thus 1000 exceeds 999 by 1; and since the largest numbers by which each of the places can be filled up is 9, unity in any place is always greater in value than all that is to the right of it.

CHAPTER II.

ON MODES OF COMPUTATION.

1. To read off numbers with fluency, group them into periods of three, making the units' figure the last figure of a period, and remembering that the lowest period of three is read off as units, the next as thousands, the next as millions, and so on; e.g. 378,894,216 will be read off 378 millions, 894 thousands, 216.

The same method of grouping quantities will be found advantageous in writing numbers from dictation.

2. Learn to count as follows, *uttering the words as fast as you can talk*:

a.	1, 2, 3, 4, 5, &c. up to	100	*k.*	1, 6, &c. up to	101
b.	1, 3, 5, 7	101	*l.*	2, 7	102
c.	2, 4, 6, 8	100	*m.*	3, 8	103
			n.	4, 9	104
d.	1, 4, 7, 10	100	*o.*	5, 10	100
e.	2, 5, 8, 11	101			
f.	3, 6, 9, 12	102	*p.*	1, 7, 13	103
			q.	2, 8, 14	104
g.	1, 5, 9	101	*r.*	3, 9, 15	105
h.	2, 6, 10	102	*s.*	4, 10, 16	100
i.	3, 7, 11	103	*t.*	5, 11, 17	101
j.	4, 8, 12	100	*u.*	6, 12, 18	102

v.	1, 8, 15, &c. up to		106
w.	2, 9, 16	......	100
x.	3, 10, 17	......	101
y.	4, 11, 18	......	102
z.	5, 12, 19	......	103
aa.	6, 13, 20	......	104
ab.	7, 14, 21	......	105
ac.	1, 9, 17	......	105
ad.	2, 10, 18	......	106
ae.	3, 11, 19	......	107
af.	4, 12, 20	......	100
ag.	5, 13, 21	......	101
ah.	6, 14, 22, &c. up to		102
ai.	7, 15, 23	......	103
aj.	8, 16, 24	......	104
ak.	1, 10, 19	......	100
al.	2, 11, 20	......	101
am.	3, 12, 21	......	102
an.	4, 13, 22	......	103
ao.	5, 14, 23	......	104
ap.	6, 15, 24	......	105
aq.	7, 16, 25	......	106
ar.	8, 17, 26	......	107
as.	9, 18, 27	......	108

The following ring will facilitate the learning of the series, for the addition of threes, sixes and sevens, viz. *d—f*, *p—u*, and *v—ab*.

8 1 4
5 7 *
2 0
9 6 3

For *d*, *e*, *f*, begin with 1, 2, 3, respectively, and read off the successive figures, travelling *with* the hands of a clock, the ring supplying the figures in the unit's place only.

For *p*, *q*, *r*, *s*, *t*, *u*, begin with 1, 2, 3, 4, 5, 6, respectively, taking every alternate figure, and again travelling *with* the hands of a clock.

For *v*, *w*, *x*, *y*, *z*, *aa*, *ab*, begin with 1, 2, 3, 4, 5, 6, 7, respectively, take every figure, but travel in the *opposite* direction to the hands of a clock.

For the addition of twos, fours, and eights, i.e. of the series *b*, *c*, *g—j*, and *ac—aj*, the two following rings can be used:

1
9 3 *
7 5

2
0 4 *
8 6

For *b* and *c*, begin with 1 and 2 respectively, and travel *with* the hands of the clock; but for these two easy series such mechanical aid is never wanted.

For *g—j*, begin with 1, 2, 3, 4, respectively, take every alternate figure, and travel *with* the hands of the clock.

* These rings ought only to be used as a stepping-stone, and should be dispensed with as soon as possible.

For *ac—aj*, begin with 1, 2, 3, 4, 5, 6, 7, 8, respectively, take every figure, and travel *against* the hands of the clock. Some learners will find it easier to remember that 8 is 10 all but 2, and will add 10 and deduct 2, dispensing with the rings.

The addition of fives will present hardly any difficulty; should any be found, teach the following series as the first step:

1, 11, 21, &c. up to	101
2, 12, 22	102
⋮ ⋮ ⋮	
10, 20, 30	100

To add nine, *ak—as*, add ten and deduct one.

In the series *ak—as*, note that the ultimate sum of the digits (separate figures) of each number equals the initial number of the series; e.g. 7, 16, 25, 34, 43, 52, 61, 70, 79, 88, 97, 106, &c. 7 = 7, 1 and 6 = 7, 2 and 5 = 7, &c.; 7 and 0 = 7, 7 and 9 = 16, but 1 and 6 again = 7, 8 and 8 = 16, and 1 and 6 = 7.

Or take *ak*: 1, 10, 19, 28, &c.:—1 = 1, 1 and 0 = 1, 1 and 9 = 10, but 1 and 0 = 1, &c.

Or take *as*: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, &c. Commencing with 9, we have 1 and 8 = 9, 2 and 7 = 9, &c.

3. *Question*. Add the digits of:—7, 9, 3, 6, 2, 4, 7, 8, 5, 1, 2, 9, 3, 7, 1, 6, 8, 5.

Answer. Ninety-three.

Q. Add aloud.

A. "7, 16, 19, 25, 27, 31, 38, 46, 51, 52, 54, 63, 66, 73, 74, 80, 88, 93."

N.B. Never allow "7 and 9 are 16, 16 and 3 are 19, 19 and 6 are 25," and so on, as this wording is fatal to rapidity.

4. Casting out Nines. Add the above series, rejecting nine as fast as it is obtained, and therefore disregarding the figure 9 itself whenever it occurs; thus: 7 (omitting 9) and 3 is 10, rejecting 9 leaves 1, and 6 is 7, and 2 is 9, rejecting this leaves 0; 4 and 7 is 11, rejecting 9 leaves 2, and 8 is 10, rejecting 9 leaves 1, and so on. *Wording*: 7, 10, 1, 7, 9, 0, 4, 11, 2, 10, 1, 6, 7, 9, 0, 3, 10, 1, 2, 8, 16, 7, 12, 3. Now it will be found that the ultimate sum of the digits of the result (in this case 93) will also be 3. That this

coincidence always holds will be proved further on, and it is a useful test of the correctness of the addition. Practise examples on §§ 3 and 4 until very great fluency is obtained. Time spent on this subject will be found a most profitable investment for the teacher, and still more for the pupil, who will after this one effort be relieved of much irksome drudgery in subsequent operations.

5. ON DOZENS. One shilling is worth twelve, or one dozen, pence, therefore every dozen articles will cost as many shillings as each article costs pence; e.g. if 1 article costs 7 pence, a dozen costs 7 shillings; if 3 pence, 3 shillings; if 9 pence, 9 shillings, and so on.

EXERCISE E.

Case I. Find the cost of a dozen things	at	2 pence each.	*Ans.*	2*s.*
,,	,,	4 pence	,,	4*s.*
,,	,,	8 pence	,,	8*s.*
,,	,,	5 pence	,,	5*s.*

Case II. Find the cost of a dozen things at 1 shilling and 2 pence each. One shilling is 12 pence, one shilling and 2 pence is 14 pence, so the dozen costs 14*s.*

EXERCISE F.

Find the cost of a dozen	things at	1*s.* 5*d.* each.	*Ans.*	17*s.*
,,	,,	1*s.* 7*d.*	,,	19*s.*
,,	,,	1*s.* 1*d.*	,,	13*s.*
,,	,,	1*s.* 8*d.*	,,	20*s.* = £1.
,,	,,	1*s.* 11*d.*	,,	23*s.* = £1. 3*s.*
,,	,,	1*s.* 9*d.*	,,	21*s.* = £1. 1*s.*

Learn by heart:

(1)

1 shilling =	12 pence.
2 shillings...	24
3	36
4	48
5	60
6	72
7	84
8	96
9	108
10	120
11	132
12	144

(2)

20 shillings =	£1.
30	£1. 10*s.*
40	£2.
50	£2. 10*s.*
60	£3.
70	£3. 10*s.*
80	£4.
90	£4. 10*s.*
100	£5.

Case III. Find the cost of a dozen things at 6*s*. each. 6*s*. is 72 pence, a dozen will cost 72*s*.; 70*s*. is £3. 10*s*., and 72*s*. is £3. 12*s*.

EXERCISE G.

Find the cost of a dozen things at	5*s*. each.	*Ans.*	£3.
,, ,,	8*s*.	,,	£4. 16*s*.
,, ,,	3*s*.	,,	£1. 16*s*.
,, ,,	4*s*.	,,	£2. 8*s*.
,, ,,	6*s*. 4*d*.	,,	£3. 16*s*.
,, ,,	7*s*. 3*d*.	,,	£4. 7*s*.
,, ,,	4*s*. 11*d*.	,,	£2. 19*s*.
,, ,,	5*s*. 10*d*.	,,	£3. 10*s*.

Case IV. Find the cost of a dozen things at 10*s*. each. 10*s*. = 120 pence, the dozen will cost 120*s*.; 100*s*. is £5; 120*s*. is £6. Similarly a dozen things at 11*s*. 4*d*. will cost £6. 16*s*., since 11*s*. = 132 pence, 11*s*. 4*d*. = 136 pence; 136*s*. = 100*s*. and 36*s*. = £5 and £1. 16*s*. = £6. 16*s*.

EXERCISE H.

Find the cost of a dozen things at	9*s*. 2*d*. each.	*Ans.*	£5. 10*s*.
,, ,,	10*s*. 3*d*.	,,	£6. 3*s*.
,, ,,	8*s*. 11*d*.	,,	£5. 7*s*.
,, ,,	11*s*. 10*d*.	,,	£7. 2*s*.
,, ,,	12*s*. 6*d*.	,,	£7. 10*s*.
,, ,,	12*s*. 11*d*.	,,	£7. 15*s*.

Case V.* If one thing costs half a penny (a half-penny), a dozen things will cost half a shilling, or 6*d*.

Find the cost of a dozen things at $3\frac{1}{2}d$. each. *Ans.* 3*s*. 6*d*., because the 3*d*. yields 3*s*. and the $\frac{1}{2}d$. 6*d*.

If one thing costs one quarter penny (a farthing), a dozen will cost one quarter shilling, or 3*d*.

Find the cost of a dozen things at $5\frac{1}{4}d$. each. *Ans.* 5*s*. 3*d*.

If one thing costs three quarter pennies (three farthings), a dozen will cost three quarter shillings, or three three-penny pieces, or 9*d*.

Find the cost of a dozen things at $8\frac{3}{4}d$. each. *Ans.* 8*s*. 9*d*.

* Any difficulty in appreciating this reasoning will be removed by exhibiting to the pupil a shilling and four three-penny pieces side by side with a penny and four farthings.

Exercise K.

Find the cost of a dozen things at	$2\frac{1}{2}d.$ each.	*Ans.*	$2s.$	$6d.$
" "	$5\frac{1}{4}d.$	"	$5s.$	$3d.$
" "	$7\frac{3}{4}d.$	"	$7s.$	$9d.$
" "	$11\frac{1}{4}d.$	"	$11s.$	$3d.$
" "	$10\frac{1}{2}d.$	"	$10s.$	$6d.$
" "	$4\frac{1}{2}d.$	"	$4s.$	$6d.$
" "	$8\frac{1}{4}d.$	"	$8s.$	$3d.$

Find the cost of a dozen things at $2s.$ $3\frac{1}{2}d.$ each. $2s. = 24d.$; $2s.$ $3d. = 27d.$; $2s.$ $3\frac{1}{2}d. = 27\frac{1}{2}d.$, therefore a dozen will cost $27\frac{1}{2}$ shillings $=$ £1. $7s.$ $6d.$

Find the cost of a dozen things at $3s.$ $8\frac{1}{4}d.$ each. $3s. = 36d.$; $3s.$ $8d. = 44d.$; $3s.$ $8\frac{1}{4}d. = 44\frac{1}{4}d.$, therefore a dozen things will cost $44\frac{1}{4}$ shillings $=$ £2. $4s.$ $3d.$

Find the cost of a dozen things at $7s.$ $1\frac{3}{4}d.$ each. $7s. = 84d.$; $7s.$ $1d. = 85d.$; $7s.$ $1\frac{3}{4}d. = 85\frac{3}{4}d.$, therefore a dozen things will cost $85\frac{3}{4}$ shillings $=$ £4. $5s.$ $9d.$

Exercise L.

	s.	d.			£.	s.	d.
Find the cost of a dozen things at	2	$7\frac{3}{4}$	each.	*Ans.*	1	11	9
" "	8	$5\frac{1}{2}$		"	5	1	6
" "	9	$10\frac{1}{4}$		"	5	18	3
" "	3	$2\frac{3}{4}$		"	1	18	9
" "	1	$11\frac{1}{2}$		"	1	3	6
" "	2	$7\frac{1}{2}$		"	1	11	6
" "	3	$6\frac{3}{4}$		"	2	2	9
" "	5	$8\frac{1}{2}$		"	3	8	6
" "	4	$1\frac{1}{4}$		"	2	9	3
" "	6	$2\frac{1}{2}$		"	3	14	6
" "	7	$3\frac{3}{4}$		"	4	7	9

6. Case VI. Learn by heart:

	s.	d.		s.	d.	
12 pence =	1	0	30 pence =	2	6	= half-a-crown.
18	1	6	36	3	0	
20	1	8	40	3	4	
24	2	0	48	4	0	

	s.	d.
60 pence =	5	0
70	5	10
72	6	0
80	6	8
84	7	0
90	7	6
96	8	0

	s.	d.	
100 pence =	8	4	
120	10	0	
150	12	6	
180	15	0	
200	16	8	
240	20	0	= one pound.

If a dozen things cost 1*s.*, each thing will cost 1*d.*; if 6*d.*, ½*d.*; if 3*d.*, ¼*d.*; and if 9*d.*, ¾*d.*

Find the cost of one thing at 9*s.* a dozen. *Ans.* 9*d.*

Find the cost of one thing at £1 a dozen. *Ans.* 1*s.* 8*d.*, because £1 = 20*s.*, therefore each article will cost 20*d.*, and 20*d.* = 1*s.* 8*d.*

Find the cost of one article at 8*s.* 6*d.* a dozen. *Ans.* 8½*d.*, because 8*s.* 6*d.* is eight and a half shillings, therefore each article costs eight and a half pence.

Find the cost of one article at 15*s.* 9*d.* per dozen. *Ans.* 1*s.* 3¾*d.*, because 15*s.* 9*d.* is fifteen and three-quarter shillings, therefore each article will cost fifteen and three-quarter pence.

Find the cost of one article at £1. 1*s.* 3*d.* a dozen. *Ans.* 1*s.* 9¼*d.*, because £1. 1*s.* 3*d.* is twenty-one and a quarter shillings, therefore each article will cost twenty-one and a quarter pence, or 1*s.* 9¼*d.*

Exercise M.

	£.	s.	d.			s.	d.
Find the cost of one thing at	0	5	9	per dozen.	*Ans.*	0	5¾
,, ,,	0	10	3		,,	0	10¼
,, ,,	0	2	6		,,	0	2½
,, ,,	0	15	6		,,	1	3½
,, ,,	1	3	9		,,	1	11¾
,, ,,	0	16	3		,,	1	4¼
,, ,,	2	5	0		,,	3	9
,, ,,	3	8	6		,,	5	8½
,, ,,	4	7	9		,,	7	3¾
,, ,,	2	1	3		,,	3	5¼
,, ,,	4	13	6		,,	7	9½
,, ,,	3	9	9		,,	5	9¾
,, ,,	1	6	3		,,	2	2¼
,, ,,	2	17	3		,,	4	9¼

	£.	s.	d.			s.	d.
Find the cost of one thing at	3	15	9	per dozen.	*Ans.*	6	$3\frac{3}{4}$
" "	4	8	6		"	7	$4\frac{1}{2}$
" "	4	18	9		"	8	$2\frac{3}{4}$
" "	3	11	6		"	5	$11\frac{1}{2}$
" "	2	14	3		"	4	$6\frac{1}{4}$

These six cases are intended to give the pupil facility in interconverting pounds, shillings, and pence. Other applications of the shillings and pence tables may be found equally serviceable, as, for instance, the calculation of interest at 5 per cent. per annum, which amounts to a payment of a shilling per pound each year, and a penny per pound each month.

Case I. Find the interest to be paid on a loan of £45 for one month. *Ans.* 3*s.* 9*d.*, because for one pound we pay one penny, therefore for £45 we pay 45 pence, or 3*s.* 9*d.*

Exercise N.

				s.	d.
Find the interest on	£70	for one month.	*Ans.*	5	10
"	50	"	"	4	2
"	66	"	"	5	6
"	94	"	"	7	10
"	100	"	"	8	4
"	37	"	"	3	1
"	120	"	"	10	0

Case II. Find the interest to be paid on a loan of £56 for one year. *Ans.* £2. 16*s.*, because for one pound we pay one shilling, therefore for £56 we pay 56 shillings, or £2. 16*s.*

Exercise O.

				£.	s.	d.
Find the interest on	£17	for one year.	*Ans.*	0	17	0
"	24	"	"	1	4	0
"	38	"	"	1	18	0
"	41	"	"	2	1	0
"	52	"	"	2	12	0
"	60	"	"	3	0	0
"	79	"	"	3	19	0
"	83	"	"	4	3	0

Exercise P.

		s.	d.
How many shillings are 14 pence?	*Ans.*	1	2
„ 21	„	1	9
„ 32	„	2	8
„ 87	„	7	3
„ 79	„	6	7
„ 53	„	4	5
„ 46	„	3	10
„ 95	„	7	11
„ 41	„	3	5
„ 12	„	1	0
„ 23	„	1	11
„ 78	„	6	6
„ 97	„	8	1
„ 35	„	2	11
„ 60	„	5	0
„ 59	„	4	11
„ 48	„	4	0

Exercise Q.

		£.	s.	d.
How many pounds are 91 shillings?	*Ans.*	4	11	0
„ 82	„	4	2	0
„ 73	„	3	13	0
„ 64	„	3	4	0
„ 55	„	2	15	0
„ 46	„	2	6	0
„ 37	„	1	17	0
„ 28	„	1	8	0
„ 89	„	4	9	0
„ 70	„	3	10	0
„ 68	„	3	8	0
„ 100	„	5	0	0
„ 115	„	5	15	0
„ 120	„	6	0	0
„ 128	„	6	8	0
„ 138	„	6	18	0
„ 150	„	7	10	0

CHAPTER III.

ADDITION.

1. CONCRETE. The word Addition is derived from the Latin *addo*, and means putting together.

2. Learn by heart: *This sign* (+) *is called* PLUS, *and means that the quantities between which it stands are to be added together.*

3. Learn by heart: *In Addition, the quantities to be added together are called the* ADDENDA, *and the answer is called the* SUM.

4. Add together £5, £4, £8, £7. *Ans.* £24.

Find the sum of 6*s.*, 5*s.*, 9*s.*, 4*s.*, 7*s.* *Ans.* 31*s.*, or £1. 11*s.*

Simplify 7*d.* + 4*d.* + 9*d.* + 6*d.* + 11*d.* *Ans.* 37 pence, or 3*s.* 1*d.*

Add together 8*s.* 3*d.* and 7*s.* 4*d.* *Ans.* 15*s.* 7*d.*

Add together 4*s.* 9*d.*, 5*s.* 4*d.*, 6*s.* 7*d.*, 9*s.* 3*d.*, and 2*s.* 8*d.* It is usual and convenient to write the addenda under one another, thus:

s.	*d.*
4	9
5	4
6	7
9	3
2	8

Let us begin with the shillings, as the more important. *Ans.* 26 shillings and 31 pence; but as 31 pence are 2*s.* 7*d.*, the handier answer is 28*s.* 7*d.*, or £1. 8*s.* 7*d.* From this it will be seen that labour would be saved by beginning with the pence, as the sum of the shillings can make no alteration in that of the pence, while the sum of the pence may yield some shillings.

Simplify 6*s.* 8*d.* + 4*s.* 3*d.* + 5*s.* 9*d.* + 3*s.* 11*d.* + 9*s.* 10*d.* + 1*s.* 2*d.*

Modus operandi:

s.	*d.*
6	8
4	3
5	9
3	11
9	10
1	2

Add up the pence column, 43 pence, which are 3*s.* 7*d.*; put down the 7*d.* under the pence, and add on or *carry* the 3*s.* to the shillings'

column, which now yields 31 shillings, or £1. 11*s.*; hence the total answer is £1. 11*s.* 7*d.*

Simplify £5. 6*s.* 0*d.* + £8. 3*s.* 0*d.* + £9. 7*s.* 0*d.* + £1. 4*s.* 0*d.* + £3. 9*s.* 0*d.* + £4. 8*s.* 0*d.*

Mod. op.:

£.	s.	d.
5	6	0
8	3	0
9	7	0
1	4	0
3	9	0
4	8	0

Add the shillings' column, which yields 37 shillings, or £1. 17*s.*; write the 17 shillings under the shillings, and *carry* the £1 to the pounds' column, which now yields £31; therefore the total answer is £31. 17*s.*

Find the sum of £6. 8*s.* 3*d.*, £2. 5*s.* 7*d.*, £3. 9*s.* 2*d.*, £5. 7*s.* 4*d.*, £9. 6*s.* 9*d.*

Mod. op.:

£.	s.	d.
6	8	3
2	5	7
3	9	2
5	7	4
9	6	9
26	17	1

Add the pence column, which yields 25 pence, or 2*s.* 1*d.*; write the 1*d.* under the pence column, and *carry* the 2*s.* to the shillings, which now yield 37 shillings, or £1. 17*s.*; write the 17 shillings under the shillings' column, and carry the pound to the pounds' column, which now yields £26; therefore the total answer is £26. 17*s.* 1*d.*, which is written under the line, as above.

Find the sum of $4\frac{3}{4}d.$, $5\frac{1}{2}d.$, $8\frac{1}{2}d.$, $9\frac{1}{4}d.$, and $10\frac{3}{4}d.$

Mod. op.:

	d.
	$4\frac{3}{4}$
	$5\frac{1}{2}$
	$8\frac{1}{2}$
	$9\frac{1}{4}$
	$10\frac{3}{4}$
3	$2\frac{3}{4}$

Add the farthings, counting the $\frac{1}{2}d.$ as two farthings; this yields 11 farthings, or $2\frac{3}{4}d.$; write the $\frac{3}{4}d.$ under the farthings, and carry

the 2*d.* to the pence, which now yield 38 pence, or 3*s.* 2*d.*; therefore the answer is 3*s.* $2\frac{3}{4}$*d.*

Add £4. 3*s.* 7*d.*, 4*s.* $5\frac{1}{2}$*d.*, £7. 8*s.* 8*d.*, $9\frac{1}{2}$*d.*, £6. 0*s.* $3\frac{1}{4}$*d.*, £9. 5*s.* $0\frac{1}{2}$*d.*, £3. 0*s.* $6\frac{3}{4}$*d.*, 8*s.* $0\frac{3}{4}$*d.*, £8. 9*s.* $4\frac{3}{4}$*d.*

Mod. op.:

£.	s.	d.
4	3	7
	4	$5\frac{1}{2}$
7	8	8
		$9\frac{1}{2}$
6	0	$3\frac{1}{4}$
9	5	$0\frac{1}{2}$
3	0	$6\frac{3}{4}$
	8	$0\frac{3}{4}$
8	9	$4\frac{3}{4}$
39	0	10

Add the farthings: 16 farthings are 4*d.*, which we carry to the pence, not writing anything under the farthings; add the pence: 46 pence are 3*s.* 10*d.*, write 10*d.* under the pence, and carry 3*s.* to the shillings; add the shillings: 40*s.* are £2 exactly, therefore write 0 under the shillings, and carry the £2; add the pounds, which now yield £39, making the total £39. 0*s.* 10*d.*

£.	s.	d.
3	7	4
5	2	$8\frac{1}{2}$
7	4	$5\frac{1}{4}$
4	8	$6\frac{3}{4}$
5	6	$1\frac{1}{2}$
4	0	$9\frac{3}{4}$
7	2	$11\frac{1}{2}$
36	12	$11\frac{1}{4}$

The following wording, *and no more*, is to be used: 2, 5, 7, 10, 11, 13 (farthings), 1, carry 3; 14, 23, 24, 30, 35, 43, 47 (pence), 11, carry 3; 5, 11, 19, 23, 25, 32 (shillings), 12, carry 1; 8, 12, 17, 21, 28, 33, 36 (pounds).

EXERCISE I.

(1)

£.	s.	d.
5	4	2
1	7	8

(2)

£.	s.	d.
5	3	4
8	2	6

(3)

£.	s.	d.
7	6	10
8	4	6

(4)

£.	s.	d.
7	8	$11\frac{3}{4}$
5	6	$7\frac{1}{2}$

(5)

9	8	4
2	7	3
6	5	7

(6)

5	4	$6\frac{1}{4}$
3	8	$9\frac{1}{4}$
1	2	$7\frac{1}{2}$

(7)

7	2	$3\frac{1}{4}$
8	4	$2\frac{1}{2}$
5	7	$6\frac{3}{4}$

(8)

1	2	3
4	5	6
7	8	9
	4	5

(9)			(10)			(11)			(12)		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
1	3	5½	7	7	7¼	3	7	4	7	6	5
5	8	9¾	8	9	10½	6	4	8¼	4	3	2½
7	5	3¼		4	5¼	4	9	2½	8	5	2
9	2	7	3	2	7	2	6	0½	9	6	9¼
									2	3	8

(13)			(14)			(15)			(16)		
8	7	9¾	1	2	4½	6	2	4½	2	6	9¾
8	7	9¾	4	8	3	6	5	9¼	3	7	11½
8	7	9¾	1	5	7¾	3	2	10½	4	8	10
8	7	9¾	5	7	7¼	7	0	0¾	5	9	9¼
8	7	9¾	9	1	7¼	8	8	8	6	5	8

(17)			(18)			(19)			(20)		
3	6	4	8	4	5	1	2	3¼	1	4	6¾
3	8	8	6	3	9	4	5	6½	9	6	7½
6	6	7	9	4	10¼	7	8	9	2	6	8¼
2	6	6	7	6	7½	9	6	3¾	6	8	4¼
9	4	1	5	5	5	6	8	5	3	6	9¼
7	9	9	3	7	8	7	4	1½	2	7	0

(21)			(22)			(23)			(24)		
4	7	7½	7	6	4¾	2	7	8	3	1	6¼
4	8	11½	8	6	3¾	5	9	3	7	4	5½
2	5	11½	5	9	4¼	4	2	11	2	7	9¾
7	8	9¼	4	7	11½	7	8	4	2	8	10¼
6	6	6¼	5	8	9¼	6	2	1	9	0	8¼
7	9	5¾	2	2	1½	1	0	6	3	4	11¾
						3	9	10	1	6	2¾

(25)			(26)			(27)			(28)		
1	3	9½	8	9	4	9	8	7½	6	7	9¼
2	7	2¾	2	5	7	6	5	4¼	1	8	10¼
3	6	4	8	6	3	3	2	1½	2	9	11½
	8	7¼	6	9	5	1	4	2¼	3	5	4¾
4	9	1¾	1	4	9	7	5	8¾	9	7	6¾
6	9	5½	3	8	10	3	8	4½	7	5	9¼
8	8	8	6	4	11	6	1	5¾	8	7	2½

(29)			(30)			(31)			(32)		
6	9	11¼	8	1	2	5	7	8	8	7	10½
4	9	4½	7	3	4	3	8	7½	5	4	4
2	6	6¾	6	5	9	7	7	9	6	2	2¼
3	9	9¾	6	5	4	2	8	9	7	9	9
8	7	2½	3	2	1	9	8	9	7	7	10¾
8	9	7¾	3	7	8	4	7	5¼	6	7	4
6	7	5½	2	1	11	5	9	4	8	6	5½
			1	4	10	8	8	3	4	3	2½

(33)			(34)			(35)			(36)		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
1	9	7¼	7	8	7½	1	9	10¼	6	8	4
2	0	8½	7	9	9	4	8	9½	6	9	5
3	1	9¾	6	0	4	2	7	11¼	7	3	10
4	2	10¼	5	0	3	3	8	2½	4	3	11
5	3	11½	9	9	8	6	9	8¾	7	7	7
6	4	1¾	7	6	3	8	7	6¾	6	8	4
7	5	2¼	2	2	0¼	4	6	7¼	3	2	7
8	6	3½	5	5	7½	7	5	3	1	1	1¼

(37)			(38)			(39)			(40)		
5	5	3	4	7	3½	1	2	3½	1	0	2
7	6	8¼	6	5	9¼	4	5	6½	2	2	5
4	9	7	8	8	10¾	7	8	9½	3	4	8½
9	9	4¼	2	2	4	1	0	11¼	4	6	11
2	3	5	1	6	11	1	2	1¾	5	8	4
6	0	7¼	3	5	5¾	4	5	6¼	6	0	7
7	7	7	5	9	1¼	5	6	7¼	7	2	0¼
8	7	7¼	7	8	2½	3	6	9¾	8	4	3
						7	6	5¾	9	6	6

(41)			(42)			(43)			(44)		
8	0	10½	6	5	7½	7	6	5½	9	9	7½
7	0	3¼		5	7½	8	8	10¼	7	6	4¼
9	0	5½	1	7	5¼	3	6	11¾	3	7	6½
6	0	7¼	7	6	11	6	8	10½	3	9	7½
2	0	8½		8	9	2	7	11¼	6	2	5¼
2	0	9½		4	8½	9	7	11¾	3	2	3
2	0	6¼	3	7	7	4	9	9½	7	6	5
3	0	6¾	2	6	8	1	4	7½	4	0	0
7	0	2¾	9	9	9	5	3	6¼	9	9	9¼

(45)			(46)			(47)			(48)		
3	2	0	9	8	7¼	3	7	8¾	6	1	5
1	8	9½	9	9	10¾	3	7	8¾	2	3	11
4	1	9¼	9	3	7¾	3	7	8¾	9	7	6
1	4	0½	5	7	8¾	3	7	8¾	8	7	10
5	2	9¾	6	6	6½	3	7	8¾	8	9	9
9	8	7	7	5	2¼	3	7	8¾	5	7	3
2	7	6½	3	0	4¼	3	7	8¾	3	4	6
1	6	3	4	2	2¼	3	7	8¾	8	6	3
4	3	2¼	2	9	11½	3	7	8¾	9	8	7
			7	6	3¾	3	7	8¾	2	3	2

(49) £.	s.	d.	(50) £.	s.	d.	(51) £.	s.	d.	(52) £.	s.	d.
1	6	11½	1	0	9¼	7	6	8½	8	5	3¼
2	7	6¾	2	8	7½	1	5	11¾	9	6	4
3	8	5¼	3	6	5¾	2	7	3	7	3	11½
4	1	7¼	6	4	11¼	3	8	5½	7	8	10
5	9	4½	4	9	10¾	8	9	8	9	5	6¾
6	2	8¾	8	7	8½	9	1	7¼	8	3	7
7	3	3¼	9	4	5½	4	4	6	8	7	7¾
8	5	9¾	3	3	2¾	6	2	2¾	9	5	9
9	4	2½	7	9	4¼	5	6	10	7	6	4½
9	0	10½	4	4	6½	5	3	8	7	8	6
						5	3	1¼	9	3	3¼
									6	2	5½

5. Add 13*s.*, 17*s.*, 19*s.*, 15*s.*, 12*s.*, 14*s.*, 16*s.*, 18*s.*

	s.
	13
	17
	19
	15
	12
	14
	16
	18
6	4

Wording: 8, 14, 18, 20, 25, 34, 41, 44 shillings, 4*s.*, carry 4 (half-sovs.); 5, 6, 7, 8, 9, 10, 11, 12 (half-sovs.), or £6. Total answer, £6. 4*s.*

Add £5. 17*s.* 0*d.*, £8. 13*s.* 0*d.*, £9. 11*s.* 0*d.*, £4. 15*s.* 0*d.*, £7. 13*s.* 0*d.*, £1. 10*s.* 0*d.*

£.	s.	d.
5	17	0
8	13	0
9	11	0
4	15	0
7	13	0
1	10	0
37	19	0

Wording: 0 pence; 3, 8, 9, 12, 19 (shillings), 9, carry 1 (half-sov.); 2, 3, 4, 5, 6, 7 (half-sovs.), 1, carry 3 (pounds); 4, 11, 15, 24, 32, 37. Total, £37. 19*s.* 0*d.*

Add £7. 18*s.* 6¾*d.*, £5. 19*s.* 10½*d.*, £3. 11*s.* 4¼*d.*, £6. 13*s.* 8¾*d.*, £5. 8*s.* 7½*d.*

£.	s.	d.
7	18	6¾
5	19	10½
3	11	4¼
6	13	8¾
5	8	7½
29	12	1¾

Wording: 2, 5, 6, 8, 11 (farthings), 3, carry 2; 9, 17, 21, 31, 37 (pence), 1, carry 3; 11, 14, 15, 24, 32 (shillings), 2, carry 3 (half-sovs.); 4, 5, 6, 7 (half-sovs.), 1, carry 3; 8, 14, 17, 22, 29. Total, £29. 12*s.* 1¾*d.*

Add £17. 14*s.* $3\frac{1}{4}$*d.*, £159. 11*s.* $5\frac{3}{4}$*d.*, £586. 16*s.* $11\frac{1}{2}$*d.*, £83. 13*s.* 2*d.*, £36. 18*s.* $10\frac{1}{2}$*d.*

£.	*s.*	*d.*
C. X. I.	£½. I.	
1 7	1 4	$3\frac{1}{4}$
1 5 9	1 1	$5\frac{3}{4}$
5 8 6	1 6	$11\frac{1}{2}$
8 3	1 3	2
3 6	1 8	$10\frac{1}{2}$
8 8 4	1 4	9

Wording: **2, 4, 7, 8, carry 2 ; 12, 14, 25, 30, 33, 9, carry 2 ; 10, 13, 19, 20, 24, 4, carry 2 ; 3, 4, 5, 6, 7, 1, carry 3 ; 9, 12, 18, 27, 34 (pounds), 4 (pounds) and carry 3 (tens of pounds) ; 6, 14, 22, 27, 28 (tens of pounds), 8 (tens of pounds), carry 2 (hundreds of pounds) ; 7, 8 (hundreds of pounds). Total, £884. 14*s.* 9*d.***

Add £5286. 16*s.* $8\frac{1}{2}$*d.*, £397. 13*s.* $10\frac{3}{4}$*d.*, £6417. 18*s.* $5\frac{1}{2}$*d.*, £96. 8*s.* $7\frac{3}{4}$*d.*, £12497. 16*s.* $8\frac{1}{2}$*d.*, £638. 3*s.* 9*d.*, £8. 7*s.* $3\frac{3}{4}$*d.*

£.	*s.*	*d.*
XM. M. C. X. I.	£½. I.	
5 2 8 6	1 6	$8\frac{1}{2}$
3 9 7	1 3	$10\frac{3}{4}$
6 4 1 7	1 8	$5\frac{1}{2}$
9 6	8	$7\frac{3}{4}$
1 2 4 9 7	1 6	$8\frac{1}{2}$
6 3 8	3	9
8	7	$3\frac{3}{4}$
2 5 3 4 3	5	$5\frac{3}{4}$

Wording: **3, 5, 8, 10, 13, 15, 3, carry 3; 6, 15, 23, 30, 35, 45, 53, 5, carry 4 ; 11, 14, 20, 28, 36, 39, 45, 5, carry 4; 5, 6, 7, 8, carry 4; 12, 20, 27, 33, 40, 47, 53 (pounds), 3, carry 5 (tens of pounds); 8, 17, 26, 27, 36, 44 (tens of pounds), 4, carry 4 (hundreds of pounds); 10, 14, 18, 21, 23 (hundreds of pounds), 3, carry 2 (thousands of pounds); 4, 10, 15, 5, carry 1 (ten thousand pounds); 2. Total, £25343. 5*s.* $5\frac{3}{4}$*d.***

Add £7486. 15*s.* 10*d.*, £2593. 13*s.* $8\frac{1}{2}$*d.*, £837. 11*s.* $10\frac{3}{4}$*d.*, £3596. 16*s.* $3\frac{1}{2}$*d.*, £38. 5*s.* $11\frac{1}{4}$*d.*, £42,376. 7*s.* $7\frac{3}{4}$*d.*, £2000, £1120. 1*s.* 10*d.*, 7*s.* 11*d.*

£.	*s.*	*d.*
7486	15	10
2593	13	$8\frac{1}{2}$
837	11	$10\frac{3}{4}$
3596	16	$3\frac{1}{2}$
38	5	$11\frac{1}{4}$
42376	7	$7\frac{3}{4}$
2000	0	0
1120	1	10
	7	11
60050	1	$0\frac{3}{4}$

Wording: **3, 4, 6, 9, 11, 3, carry 2; 13, 23, 30, 41, 44, 54, 62, 72, 0, carry 6; 13, 14, 21, 26, 32, 33, 36, 41, 1, carry 4; 5, 6, 7, 8, carry 4; 10, 18, 24, 31, 34, 40, 0, carry 4; 6, 13, 16, 25, 28, 37, 45, 5, carry 4; 5, 8, 13, 21, 26, 30, 0, carry 3; 4, 6, 8, 11, 13, 20, 0, carry 2; 6. Total, £60,050. 1*s.* $0\frac{3}{4}$*d.***

Before dropping the cumbrous heading and wording adopted in the earlier cases, *several* sums should be worked *aloud*, till the pupil is perfectly familiar with the meaning of each column.

EXERCISE II.

(1)

£.	s.	d.
538	13	10½
8427	15	8¾
13	17	11
642	8	7½

(2)

£.	s.	d.
7040	19	1½
86	13	5¾
942	8	7¼
2568	10	10½

(3)

£.	s.	d.
42768	7	8½
13590	13	2¾
276	12	1¼
8402	5	8¾
35679	16	3¾

(4)

796	15	8¾
1248	19	11
2125	13	10¾
203	4	3¼
8751	0	1
2125	13	10¾

(5)

7684	7	8½
	15	3¼
19	13	10
256	15	11¾
1825	8	4¼
32769	12	9½
82103	1	7

(6)

786	13	10
5419	12	8
8	17	6
427	11	2
2040	13	3
966	16	6
7045	1	11

(7)

2197	12	10¼
1208	0	8½
319	14	11¾
9420	9	9¾
8531	16	7½
7642	1	2¼
6753	18	6¾
5864	3	5½

(8)

127556	11	4½
71042	2	3
48931	19	7¼
1632197	3	6½
49823	18	9¾
3286	5	8
254719	16	10½
6609	7	11½

(9)

416	13	11
5274	14	6
3708	5	8
2415	9	9
56780	17	3
26317	6	4
8412	18	10
90710	4	1
11768	12	2
1259	17	5
32708	8	7
673591	19	6

(10)

1678	18	11½
5246	17	10
356	13	5¾
8562	5	9
3427	6	0½
6	11	5¼
249	12	3½
8542	6	0¼
67	10	8¼
243	19	2½
14274	8	4¾
57343	9	10¾

By Dictation, if convenient:

(11) Add £539. 15*s.* 7*d.*, £18. 9*s.* 2¾*d.*

(12) Add £467. 10*s.* 9½*d.*, £8414. 3*s.* 7½*d.*, £6. 6*s.* 6½*d.*

(13) Add £558. 4*s.* 9¾*d.*, £25. 15*s.* 2¾*d.*, £532. 9*s.* 7*d.*, £7010. 11*s.* 4½*d.*

(14) Add £4279. 13*s.* 8½*d.*, £176. 15*s.* 9*d.*, £2040. 11*s.* 10¼*d.*, £1857. 16*s.* 9½*d.*, £855. 5*s.* 5¾*d.*

(15) Add £853. 12*s.* 9*d.*, £1866. 4*s.* 10*d.*, £851. 2*s.* 11*d.*, £2825. 8*s.* 4*d.*, £76,902. 11*s.* 3*d.*, £16,700. 19*s.* 11*d.*

(16) Add £9210. 3*s.* 7¾*d.*, £8127. 1*s.* 0*d.*, £8888. 0*s.* 11½*d.*, £53. 13*s.* 8½*d.*, £12,072. 3*s.* 1*d.*, £978. 16*s.* 3¼*d.*, £4063. 15*s.* 6½*d.*

(17) Add £4605. 1*s.* 10½*d.*, £2031. 15*s.* 3¾*d.*, £26,664. 12*s.* 10½*d.*, £161. 1*s.* 1½*d.*, £4024. 7*s.* 0¼*d.*, £2914. 4*s.* 4¼*d.*, £10,507. 17*s.* 7¾*d.*, £31,523. 12*s.* 11¼*d.*

(18) Add £1583. 11*s.* 3*d.*, £260. 2*s.* 4*d.*, £937. 17*s.* 10*d.*, £7064. 5*s.* 8*d.*, £525. 19*s.* 2*d.*, £348. 6*s.* 1*d.*, £69. 13*s.* 9*d.*, £8708. 4*s.* 11*d.*, £929. 18*s.* 7*d.*

(19) Add £91,021. 10*s.* 8¼*d.*, £28,943. 17*s.* 6¾*d.*, £3790. 2*s.* 4*d.*, £868. 15*s.* 9½*d.*, £5075. 9*s.* 10½*d.*, £6754. 16*s.* 5½*d.*, £538. 7*s.* 11*d.*, £4213. 12*s.* 2¾*d.*, £1967. 5*s.* 3*d.*, £2064. 13*s.* 7¼*d.*

(20) Add 63,185. 12*s.* 11½*d.*, £15,384. 10*s.* 8*d.*, £0. 8*s.* 10¼*d.*, £393,708. 6*s.* 5*d.*, £32,809. 4*s.* 3¾*d.*, £47. 2*s.* 4½*d.*, £27,512. 19*s.* 2½*d.*, £1760. 13*s.* 6*d.*, £3610. 7*s.* 7¾*d.*, £39,370. 11*s.* 1¾*d.*, £621,382. 5*s.* 0½*d.*, £3,531,658. 9*s.* 9¾*d.*

6. Abstract.

Q. Add £5, £8, £7, £10.

A. £30.

Q. Add 5 sheep, 8 sheep, 7 sheep, 10 sheep.

A. 30 sheep.

Q. Add 5 things, 8 things, 7 things, 10 things.

A. 30 things.

Teacher. Therefore we say $5 + 8 + 7 + 10$ is 30, regardless of the name of the things counted. If the name is given with the number, the number is said to be *concrete;* if the number alone is given, it is said to be *abstract;* thus, 7 sheep, £9, 8 yards, 2 sounds, 3 ideas, 5 emotions, 4 times, are all *concrete* numbers; but the num-

bers 7, 9, 8, 2, 3, 5, 4, standing alone, are *abstract*. When in Ch. I. § 12, we said that 1000 = 10 times 100, &c., we were already using abstract numbers.

Q. Simplify 7 + 9.

A. 16.

Q. What do you mean by saying that 7 + 9 = 16?

A. 7 things of one kind added to 9 things *of the same kind* make 16 things of that kind.

Q. What are 7 horses and 9 paving-stones?

A. 7 horses and 9 paving-stones.

Teacher. We see, then, that we can only *add* quantities of the same kind.

Add 43, 178, 5297, 62,045, 19, 8, 7684, 5760, 112, and 7000.

Mod. op.:

```
   43
  178
 5297
62045
   19
    8
 7684
 5760
  112
 7000
-----
88146
```

Wording: 2, 6, 14, 23, 28, 35, 43, 46′, carry 4 (lay stress on this 6, and write it down in the act of pronouncing it); 5, 11, 19, 20, 24, 33, 40, 44′, carry 4; 5, 12, 18, 20, 21′, carry 2; 9, 14, 21, 23, 28′, carry 2; 8′.

7. Test of accuracy by casting out nines. (See Ch. II. § 4.)

Add 158, 6424, 543, 12764, 8248, 1251.

158	5
6424	7
543	3
12764	2
8248	4
1251	0
29388	3′

Cast out nines from each of the addenda, writing the remainders to the right of the vertical line, as above; cast out nines from these remainders, and also from the answer of the sum; if the results do

not agree, the answer cannot possibly be correct; but if they do agree, the chances are at least 8 to 1 in favour of its correctness.

For the reason of this process, see Ch. XI. § 9.

Wording: 1, 6, 14, **5′**; 6, 10, 1, 3, **7′**; 5, 9, 0, **3′**; 1, 3, 10, 1, 7, 11, **2′**; 8, 10, 1, 5, 13, **4′**; 1, 3, 8, 9, **0′**; 4, 6, 9, 0, 7, 12, *three;* 2, 5, 13, 4, 12, *three.*

Exercise III.

(1)	(2)	(3)	(4)
1768	2416	4907	8947
94	80519	356	397
550	9743	4520	8276
		38271	50703

(5)	(6)	(7)	(8)
4210	98376	650234	78903
349	4297	152467	4782
5827	376	24901	194763
631	4788	933378	4936
7856	76847	8426	531
		151	74267

(9)	(10)	(11)	(12)
265168	142857	9	48793561
521797	428571	87	20907528
293622	285714	654	7841
370448	857142	3210	25
452451	571428	98765	3069
538196	714285	432109	97856
554303	142857	8765432	867924
		10987654	1250708

(13)	(14)	(15)	(16)
987654321	9518581	16470431	8463243
987654321	829326	30	773904
987654321	70394	7180359	58654
987654321	6207	21642	327760
987654321	41574	85	9983
987654321	536342	3617	41732044
987654321	6019483	706596	75783
987654321	75620	482575	245477
987654321	8179	8429	3685473
		23869397	266599

(17)	(18)	(19)	(20)
8376214	6142	310531	8476231
5976414	250	973572	763249
635523	2035	861627	35604
46877	367	740537	9094
39256479	47498	810453	47776
99783	5809	168179	392448
246804	597	620438	7553210
8975578	196	975162	14578239
342429	6071	289705	278547
81660	38276	496561	6298
73431	3148	424280	17844
		983276	12345678

Exercise IV.

(1) John played at marbles and won on Monday 43 marbles, on Tuesday 101, on Wednesday 8, on Thursday 19, on Friday 119, and on Saturday 50. How many did he win in the week?

(2) In a certain school there are 67 boys, 59 girls, and 111 infants. How many pupils are there?

(3) The Books of Moses consist of 187 chapters, the Histories of 226, the Prophecies of 273, Job of 42, and the writings of David and Solomon of 201. How many chapters are there in the Old Testament?

The New Testament contains 260 chapters. How many chapters are there in the whole Bible?

(4) The battle of Thermopylæ took place 490 B.C., that of Bunker's Hill 1775 A.D. Find the time intervening between the two?

(5) How much money do I require to pay the following bills: butcher, £23. 7*s.* 6*d.*; baker, £9. 18*s.* 10*d.*; greengrocer, £1. 5*s.* 9*d.*; grocer, £7. 13*s.* 2*d.*; milkman, £2. 11*s.* 3*d.*; tailor, £15. 10*s.*; shoemaker, £7. 8*s.* 9*d.*; stationer, £4. 7*s.* 11*d.*; wine merchant, £8. 15*s.*?

(6) A cashier begins January with £48. 10*s.* 10*d.* in his till; there is paid him £75 in January, £120. 16*s.* 4*d.* in each of the next three months. How much will he then have to account for?

(7) A person left £3619. 6*s.* 8*d.* to each of his six children. Find the amount received by all of them.

(8) A farmer sold 2 oxen for £45. 12*s.* 6*d.*, a calf for £7. 15*s.*, 2 pigs for £12. 12*s.*, 4 sheep for £10, and 3 lambs at 17*s.* 9*d.* each. How many animals did he bring to market, and for how much did he sell them?

(9) A has 43 oxen, 145 sheep, 31 cows, and 19 horses; B has 57 lambs, 215 sheep, 8 horses, 7 pigs, and 10 calves; C has 60 lambs, 22 oxen, 89 sheep, and 12 calves; D has 67 cows, 28 horses, 11 pigs, and 3 lambs; E has 11 horses, 5 calves, 10 oxen, and 25 cows; F has 100 lambs, two herds of 37 oxen each, two flocks of sheep, one of 93 and the other of 39, and 50 pigs. How many animals are there of each kind, and how many altogether?

Exercise V.

(1)

£.	*s.*	*d.*
134	6	6
232	1	$11\frac{1}{4}$
1067	17	$9\frac{1}{2}$
4032	12	1
9416	9	$8\frac{1}{2}$
1067	13	8
1279	8	$7\frac{3}{4}$
4610	3	$2\frac{1}{4}$
752	15	$3\frac{1}{2}$
7187	10	$3\frac{1}{2}$
9312	8	$5\frac{1}{2}$

(2)

£.	*s.*	*d.*
13257	8	$11\frac{1}{2}$
3276	5	$9\frac{1}{4}$
46	3	$6\frac{3}{4}$
1287	14	$7\frac{1}{2}$
4917	10	8
147	0	$6\frac{3}{4}$
360	5	5
1379	17	$2\frac{3}{4}$
9	9	$10\frac{1}{2}$
1340	16	$9\frac{1}{4}$
906	10	$7\frac{1}{2}$
2222	5	1

(3)

73191211
31442376
16310732
6904109
432173041
92761528
13971140
105633198
78042
259766

(4)

5497530
5811756
761110
4028211
77436917
14376215
29351528
15286408
190589374
7319631
6911414
5016436
56616
1189401
7116917

CHAPTER IV.

SUBTRACTION.

1. The word Subtraction is derived from the Latin words *sub* and *traho*, and means drawing or taking away;—[cf. *summoveo*, to clear (a court).]

2. Learn by heart: *This sign* (—) *is called* MINUS, *and means that the quantity following it is to be subtracted or taken away from the quantity before it.*

3. Learn by heart: *In Subtraction, the quantity from which we subtract is called the* MINUEND, *the quantity to be subtracted is called the* SUBTRAHEND, *and the Remainder is called the* DIFFERENCE; e.g. if £7 be taken away from £11 the Remainder will be £4. Here £11 is the Minuend, £7 the Subtrahend, and £4 the Difference.

4. Take £8 from £13. *Ans.* £5.
Take 3*s.* 9*d.* from 7*s.* 11*d.*
Write them as in Addition.

$$\begin{array}{rr} 7 & 11 \\ 3 & 9 \\ \hline \end{array}$$

First take 3*s.* from 7*s.*, there remains 4*s.*, and the sum will so far stand thus:

$$\begin{array}{rr} 7 & 11 \\ 3 & 9 \\ \hline 4 & \end{array}$$

Now take 9*d.* from 11*d.*, there remains 2*d.*, so that the whole difference is 4*s.* 2*d.*

$$\begin{array}{rr} 7 & 11 \\ 3 & 9 \\ \hline 4 & 2 \end{array}$$

Take 6*s.* 8*d.* from 9*s.* 5*d.*:

$$\begin{array}{rr} 9 & 5 \\ 6 & 8 \\ \hline \end{array}$$

First take 6*s.* from 9*s.*, there remains 3*s.*, and the sum will so far stand thus:

$$\begin{array}{rr} 9 & 5 \\ 6 & 8 \\ \hline 3 & \end{array}$$

Q. Next take 8*d.* from 5*d.*

A. It can't be done.

Q. If I had 9*s.* 5*d.* in my purse, could I pay 6*s.* 8*d.*?

A. Yes; because 9*s.* 5*d.* is more than 6*s.* 8*d.*

Q. After paying the 6*s.*, what will be left in my purse?

A. 3 shillings and 5 pence.

Q. How can I then pay the 8*d.*?

A. By changing one of the shillings into 12 pence.

Q. How many shillings will this now leave me?

A. Only 2 shillings.

Q. And how many pence?

A. 17 pence.

Q. Can I now pay the 8*d.*?

A. Yes.

Q. And how many pence will be left?

A. 9*d.*

Q. How much then is left altogether?

A. 2*s.* 9*d.*

Teacher. Thus the sum now stands:

$$\begin{array}{cc} 9 & 5 \\ 6 & 8 \\ \hline \cancel{3} & 9 \\ 2 & \end{array}$$

From £8. 5*s.* 1*d.* take £3. 7*s.* 9*d.*

$$\begin{array}{ccc} 8 & 5 & 1 \\ 3 & 7 & 9 \\ \hline 5 & & \end{array}$$

From £8 take £3, there remains £5.

Q. Next take 7*s.* from 5*s.*

A. It can't be done.

Q. How then shall I pay the 7*s.*?

A. Change one of the 5 pounds into 20 shillings.

Q. How many pounds will that leave me?

A. 4 pounds.

Teacher. Thus the sum now stands :

$$\begin{array}{rrr} \pounds 8 & 5 & 1 \\ 3 & 7 & 9 \\ \hline \cancel{5} & & \\ 4 & & \end{array}$$

Q. How many shillings shall I now have in my purse ?

A. 25 shillings.

Q. If I pay the 7 shillings, how many shillings will be left ?

A. 18 shillings.

Teacher. Thus the sum now stands :

$$\begin{array}{rrr} \pounds 8 & 5 & 1 \\ 3 & 7 & 9 \\ \hline \cancel{5} & 18 & \\ 4 & & \end{array}$$

Q. Now pay 9*d.*

A. Change one of the 18 shillings.

Q. How many shillings will remain ?

A. 17 shillings.

Teacher. The sum now stands :

$$\begin{array}{rrr} \pounds 8 & 5 & 1 \\ 3 & 7 & 9 \\ \hline \cancel{5} & \cancel{18} & \\ 4 & 17 & \end{array}$$

Q. How many pence have I now ?

A. 13 pence.

Q. Pay 9*d.*; remainder ?

A. 4*d.*

Q. What then is the whole remainder ?

A. £4. 17*s.* 4*d.*

Teacher. The sum now stands :

$$\begin{array}{rrr} \pounds 8 & 5 & 1 \\ 3 & 7 & 9 \\ \hline \cancel{5} & \cancel{18} & 4 \\ 4 & 17 & \end{array}$$

∴* the difference between £8. 5*s.* 1*d.* and £3. 7*s.* 9*d.* is £4. 17*s.* 4*d.*

* ∴ = therefore.

Subtract £48. 17*s.* 9¾*d.* from £72. 3*s.* 11½*d.*

£.		*s.*		*d.*
X.	I.	£½.	I.	
7	2		3	11½
4	8	1	7	9¾
$\cancel{3}$	$\cancel{4}$	$\cancel{1}$	6	$\cancel{2}$¾
2	3			1

First pay 4 ten-pound notes out of 7 ten-pound notes; there remain 3 ten-pound notes; next we have to pay £8 out of £2, for which we must change one of the 3 ten-pound notes, leaving only 2 ten-pound notes. The ten-pound note changed, together with the £2 we had at first, yield £12, and paying £8, we have left £4. Next we have to pay one half-sovereign out of none, for which we must change one of the 4 pounds, leaving only £3; that £1 yields 2 half-sovereigns, and paying one of these leaves us 1 half-sovereign. Next we have to pay 7 shillings out of 3 shillings, for which we must change the half-sovereign; and this, together with the 3*s.* we had at first, yields 13*s.*; paying 7*s.* out of it, we have left 6*s.* Next we have to pay 9*d.* out of 11*d.*, and we have 2*d.* left. Lastly, we have to pay 3 farthings out of a halfpenny, for which we must change one of the 2 pence, leaving only 1*d.*, and yielding, with the halfpenny we had at first, 6 farthings, out of which we can pay 3 farthings, and have 3 farthings left. The total difference accordingly is £23. 6*s.* 1¾*d.**

This method of subtraction is always applicable; but to avoid erasures, it is better to begin with the lowest instead of the highest denomination. Thus:

£.		*s.*		*d.*
X.	I.	£½.	I.	
7·	2·	·	3	11·½
4	8	1	7	9 ¾
2	3		6	1 ¾

First pay ¾*d.* out of ½*d.*; it cannot be done; therefore change 1*d.*

* The object of this method of teaching is to lead the pupil along some such road as must have been traversed by the original inventors of the different Rules in arriving at the most concise processes, thus attaching the new notions to be acquired to those previously existing in the mind of the learner, or, in other words, *educing* the new ideas out of old ones, which is true *education*.

of the $11d.$, leaving $10d.$, and, to remind ourselves that we have done so, make a dot by the side of the $11d.$; this penny changed, together with the halfpenny we had at first, yields 6 farthings, out of which we pay $\frac{3}{4}d.$ and have $\frac{3}{4}d.$ left. Next pay $9d.$ out of $10d.$, which leaves $1d.$ Next pay 7 shillings out of 3 shillings; to do so, change one of the 2 pounds (putting the dot by the two) into 2 half-sovereigns; leave one of these in the half-sovereign column, and indicate by the dot that the other has been taken away and changed into $10s.$; this, together with the $3s.$, yields $13s.$; pay the $7s.$, there remain $6s.$ Next pay one half-sovereign, and no half-sovereigns are left. Next pay £8 out of £1, for which change one of the 7 ten-pound notes, leaving 6, and yielding, together with the £1, £11. Now pay the £8, leaving £3. Lastly, pay the 4 ten-pound notes out of the 6, and 2 ten-pound notes are left. Total remainder or difference, as before, £23. $6s.$ $1\frac{3}{4}d.$

From £1000, take £37. $8s.$ $6d.$

£.				$s.$		$d.$
M.	C.	X.	I.	£½.	I.	
1·	0·	0·	0·	·	0·	0
		3	7		8	6
	9	6	2	1	1	6

$6d.$ cannot be taken from $0d.$, therefore we must change some higher coin or note. The only money we have is 1 thousand-pound note, which we change into 10 hundreds, putting a dot as before after the 1; we take now one of these 10 hundreds to the tens, leaving 9 hundreds in the hundreds' column, which is again indicated by a dot; this hundred gives ten tens, of which we take one, leaving 9 in the tens' column; this ten gives £10, of which we take £1, leaving 9 in the pounds' column; this £1 gives 2 half-sovereigns, of which we take one, leaving 1 under the half-sovereign column; the 1 we take gives 10 shillings, of which we take 1 shilling and leave 9; this shilling gives 12 pence. $6d.$ from $12d.$ leaves $6d.$; $8s.$ from $9s.$ leaves $1s.$; nothing from 1 half-sovereign leaves 1 half-sovereign; £7 from £9 leaves £2; 3 ten-pound notes from 9 ten-pound notes leaves 6 ten-pound notes; and nothing from 9 hundred-pound notes leaves 9 hundred-pound notes. Total remainder, £962. $11s.$ $6d.$

EXAMPLES.

£.	s.	d.	Wording:
8	7	9	6 from 9, 3′
1	5	6	5 ,, 7, 2′
7	2	3	1 ,, 8, 7′

£.	s.	d.	Wording
£8·	·5·	3	7 from 15 (.) 8
3	9	7	9 ,, 14 (..) 5
4	15	8	0 ,, 1 1
			3 ,, 7 4

£.	s.	d.	Wording
£4·2 9·	1·3·	2·¼	3 from 5 (.) ½d.
1 7 6	1 8	9 ¾	9 ,, 13 (.) 4
2 5 2	1 4	4 ½	8 ,, 12 (.) 4
			1 ,, 2 (.) 1
			6 ,, 8 2
			7 ,, 12 (.) 5
			1 ,, 3 2

£.	s.	d.	Wording
£6·0·5·0·	4·	0·	1 from 4 (..) 3
1 7 6 8	1 2	2 ¼	2 ,, 11 9
4 2 8 1	1 1	9 ¾	2 ,, 3 1
			1 ,, 2 (..) 1
			8 ,, 9 1
			6 ,, 14 (..) 8
			7 ,, 9 2
			1 ,, 5 4

£.	s.	d.	Wording
£2 7·0·0·	0·	0	7 ,, 12(....)5
1 8 3	3	7	3 ,, 19* 16
2 5 1 6	1 6	5	3 ,, 9 6
			8 ,, 9 1
			1 ,, 6 5
			0 ,, 2 2

From £87,694. 0*s.* 0*d.*, subtract £24,132. 0*s.* 0*d.*

£.	s.	d.	Wording:
8 7 6 9 4	0	0	2 from 4, 2
2 4 1 3 2	0	0	3 ,, 9, 6
6 3 5 6 2	0	0	1 ,, 6, 5
			4 ,, 7, 3
			2 ,, 8, 6

* Where there are no half-sovereigns in the minuend, it is better to call £1 20 shillings, instead of 2 half-sovereigns.

From £42,618, take £16,942.

4·2·6·1 8	*Wording:*	2	from	8	6
1 6 9 4 2		4	,,	11 (.)	7
2 5 6 7 6		9	,,	15 (.)	6
		6	,,	11 (.)	5
		1	,,	3	2

From 90,017 units, take 18,659 units.

9·0·0·1·7	*Wording:*	9	from	17 (.)	8
1 8 6 5 9		5	,,	10 (...)	5
7 1 3 5 8		6	,,	9	3
		8	,,	9	1
		1	,,	8	7

From 1,000,000, take 63,497.

1·0·0·0·0·0·0	*Wording:*	7	from	10 (......)	3
6 3 4 9 7		9	,,	9	0
9 3 6 5 0 3		4	,,	9	5
		3	,,	9	6
		6	,,	9	3
		—		—	9

5. Test of accuracy by casting out nines. (See Ch. II. § 4, and Ch. III. § 7.)

143549	8	*Wording:* 1, 5, 8, 13, 4, **8′**; 3, 10, 1, 2, 10, **1′**; 1 from 8, *seven;* 1, 2, 4, 12, 3, 6, *seven.*
30718	1	
112831	**7′**	

1030412	2	*Wording:* 1, 4, 8, 9, 0, **2′**; 7, 13, 4, 8, 13, 4, 10, 1, **5′** (now as 5 cannot be taken from 2, add 9 to the 2, making 11); 5 from 11, *six;* 2, 8, 13, 4, 12, 3, 7, 15, *six.*
764564	5	
265848	**6′**	

314254	1	*Wording:* 3, 4, 8, 10, 1, 6, 10, **1′**; 1, 3, 6, 10, 1, 6, 12, **3′**; 3 from 10, *seven;* 1, 8, 16, *seven.*
123456	3	
190798	**7′**	

4680135	0	*Wording:* 4, 10, 1, 9, 0, 1, 4, 9, **0′**; 3, 5, 6, 12, 3, 8, 12, **3′**; 3 from 9, *six;* 1, 5, 11, 2, 5, 10, 1, *six.*
3216540	3	
1463595	**6′**	

EXERCISE VI.

	(1)			(2)	
£.	s.	d.	£.	s.	d.
778	4	5	9999	19	11¾
536	2	4	9876	5	4¾

	(3)			(4)	
46596	17	10	8712	15	3
13282	12	4	930	8	7

	(5)			(6)	
4897	7	7	8713	18	9¼
1624	8	11	1784	19	11¾

	(7)			(8)	
7638	14	3½	72312	5	9¾
147	17	4½	16743	8	10½

	(9)			(10)	
9472	4	5¾	81403	15	7½
8763	15	4¼	68	0	11

	(11)			(12)	
346004	1	2½	49060	12	1¾
76802	14	0¼	893	15	3¾

	(13)			(14)	
130764	11	3	40076	0	5¾
68943	13	6½	16307	9	8¼

	(15)			(16)	
70	10	0½	382	0	0¼
6	16	8¾	187	0	11½

	(17)			(18)	
1000	0	0	53761	0	0
999	19	11¾	7777	7	7

	(19)			(20)	
500	0	0	8000	0	0
367	18	10¼	796	17	2½

EXERCISE VII.

(1)	(2)	(3)
734968	625137	435768
123454	214021	123989

(4)	(5)	(6)
243617	310703	57345
76329	25844	16462

(7)	(8)	(9)
3467825	3087902	1000000
178062	396999	999999

(10)	(11)	(12)
376462	857142	401050
84752	428571	123456

(13)	(14)	(15)
876543	543645	283461
777777	137968	99999

(16)	(17)	(18)
194580	422060	1147368
63245	234681	654891

(19)	(20)
5000807	100100
283876	11111

Exercise VIII.

By Dictation, if possible:

(1) Take 196,734 from 321,708; also £4105. 13*s.* 8¾*d.* from £12,019. 16*s.* 2½*d.*

(2) Find the difference between 1,178,458 and 1,780,412; also between £50,100. 2*s.* 2*d.* and £16,872. 13*s.* 10½*d.*

(3) Find the excess of £105. 3*s.* 0¼*d.* over £83. 16*s.* 9½*d.*; also of 7015 over 2406.

(4) How much must I add to £13,502. 9*s.* 3*d.* to get £40,000; also to 62,318 to get 100,000?

(5) What must I take away from £11,111. 11*s.* 1¼*d.* to leave £3805. 16*s.* 2½*d.*; also from 1,010,101 to leave 909,909.

(6) The latitude of London is $51\frac{1}{2}^0$ N.; that of the Tropic of Cancer is $23\frac{1}{2}^0$ N. How many degrees is London north of this Tropic?

(7) If I gain £37,105. 8*s.* 3½*d.* and lose £50,602. 9*s.* 10*d.*, how much shall I have gained or lost altogether?

(8) In a certain school there are 102 pupils, of whom 67 are boys and the remainder girls. How many girls are there?

(9) A father is 42 years of age, and his eldest son was born when he was 26 years old. How old is the son now? Also, by how

much is the father older than the son, and how much older than the son will the father be 12 years hence?

(10) It is now eleven o'clock a.m. What o'clock was it 6, 12, 15, and 24 hours ago?

6. To how many persons can I give 8*s.* 3*d.* if I have £2. 1*s.* 3*d.*?

£2	1	3	
	8	3	1
1	13	0	
	8	3	1
1	4	9	
	8	3	1
	16	6	
	8	3	1
	8	3	
	8	3	1
	—	—	5 persons.

To how many persons can I give £53. 12*s.* 9½*d.* if I have £492. 8*s.*?

£492	8	0	
53	12	9½	1
438	15	2½	
53	12	9½	1
385	2	5	
53	12	9½	1
331	9	7½	
53	12	9½	1
277	16	10	
53	12	9½	1
224	4	0½	
53	12	9½	1
170	11	3	
53	12	9½	1
116	18	5½	
53	12	9½	1
63	5	8	
53	12	9½	1
9	12	10½	9 persons, and £9. 12*s.* 10½*d.* over.

EXERCISE IX.

(1) To how many persons can I give £19. 16*s.* 3¾*d.* each, if I have £138. 14*s.* 2¼*d*?

(2) How many shares can I buy for £1120. 10*s.*, if each costs £93. 7*s.* 6*d.*?

(3) How many times can I subtract £8. 9*s.* 4*d.* from £67. 14*s.* 8*d.*?

(4) How many heaps of 56 marbles each can I make out of 500 marbles?

(5) If I travel 144 miles a day, how many days shall I take to travel 1008 miles?

(6) How many strips of carpet, each 17 yards long, can I get out of 160 yards of carpet?

EXERCISE X.

Miscellaneous Examples on the preceding chapters.

(1) *a.* Read off: LVIII, LXIX, CXLVII, CCXCIV, MXC, MDCCCIV, MDXIX, MC, DCCCCIX, MDCCCLXIX.

b. Write in Roman Notation: Six; four; eleven; twenty-nine; one hundred and forty-four; six hundred and sixty-six; one thousand, two hundred and two; eleven hundred and fourty-four; twelve hundred and ninety-nine; one thousand, two hundred and thirty-four.

(2) *a.* Read, or write in words: 1008; 13015; 7012009; 4226843; 60606060; 987654321; 42105; 24150; 116001; 110061.

b. Write with Arabic Notation in the Decimal Scale: Five thousand and forty; twelve thousand and twelve; twelve hundred and twelve; twelve thousand, twelve hundred and twelve; thirteen millions, fourteen thousand and fifteen; fifty-eight millions; twenty millions, three hundred and sixty-five thousand and nineteen; fifty thousand and fifty; five hundred thousand and fifty; one million, one thousand and ten.

(3) Analyse (aloud or in writing) 3052604.

(4) Alfred the Great died at the age of 52, A.D. 901. In what year was he born?

(5) William the Conqueror began to reign A.D. 1066, and reigned 21 years. In what year did he die?

(6) A travels northwards 203 miles, B travels 167 miles in the same direction. How far will they be apart?

(7) If B had travelled southwards, how far apart would they be?

(8) I am now (A.D. 1869) 37 years old, and the battle of Trafalgar happened 27 years before I was born. In what year was it fought?

(9) Add the sum of 1008 and 639 to their difference.

(10) Take the difference of 1001 and 999 from their sum.

(11) A cashier receives on Monday, £596. 13*s.* 8*d.*; on Tuesday, £932. 11*s.* 4*d.*; on Wednesday, £403. 6*s.* 4*d.*; on Thursday, £67. 8*s.* 8*d.*; on Friday, £145. 17*s.* 6*d.*; and on Saturday, £854. 2*s.* 6*d.* His expenditure is on Monday, £139. 19*s.* 11*d.*; on Tuesday, £369. 8*s.* 10*d.*; on Wednesday, £860. 0*s.* 1*d.*; on Thursday, £632. 11*s.* 2*d.*; on Friday, nothing; on Saturday, £319. 9*s.* 9*d.* Find the balance in hand at the end of each day of the week.

(12) My income is £500 a-year. I spend £14. 3*s.* 6*d.* each quarter for rent; £3. 10*s.* 10½*d.* each quarter for taxes; £9. 3*s.* 4*d.* each half-year for insurance; £156 per annum for food, &c.; £7. 4*s.* for coals during the winter, and £1. 16*s.* during the summer; and £33. 6*s.* 8*d.* a-year schooling for *each* of three children. How much a-year is left me for other purposes?

(13) If I spend £2. 17*s.* 4*d.* a-week, having £16 on leaving home, how much shall I bring back after 5 weeks' holiday?

(14) If I spend every week 15*s.* 9*d.* for lodging; £1. 1*s.* for board; 4*s.* 6*d.* for travelling; 8*s.* 4*d.* for sundries; how long will £15 last me, and how much shall I be in debt at the end of 7 weeks?

(15) How many months of 30 days each are there in the 365 days of the year?

(16) An army consisted of 50,000 soldiers before the battle. The list of casualties was as follows: killed, 3,768 men and 419 officers; wounded, 9,483 men and 2,716 officers; missing, 802 men and 1 officer. What was the strength of the army after the battle?

(17) On the 1st of January I bought goods for £50. 18*s.* 6*d.*, and paid £10. 10*s.* on account. The balance is to be paid off in monthly instalments of £5. 15*s.* 6*d.* each. On what day shall I pay the last instalment?

(18) A clock and a watch are set at noon on Monday; the clock gains 4 minutes in each of its days (of 24 hours), the watch loses 2 minutes in the same time; what time will the watch indicate on Saturday when it is noon by the clock?

(19) If the clock gains 7 minutes and the watch gains 11 minutes each day by the clock, what time will the watch indicate on Saturday when it is noon by the clock?

(20) If I have £80 in the bank, and put by £52. 10*s.* a year, how long shall I be in accumulating £500?

(21) How many times must 6798 be added to 9212 to make 50,000?

(22) I have the following bills to pay: butcher, £14. 7*s.* 11*d.*; baker, £5. 9*s.* 8*d.*; grocer, £9. 10*s.* 10*d.*; greengrocer, 17*s.* 7*d.*; milkman, £1. 2*s.* 3*d.* How much shall I have left out of £35?

(23) A house with fixtures and furniture is bought for £1050; the price of the furniture is £335, that of the fixtures is £27. 10*s.* What was the sum paid for the house?

(24) John and Tom play at marbles; John begins with 158 marbles, and Tom with 271; Tom loses 56 marbles. Which has more, and by how much?

(25) At an election there were three polling-places. At the first, the Whig candidate obtained 766 votes, and the Tory 695; at the second, the Tory had 523 and the Whig 419; at the third, the Whig had 812 and the Tory 811. Which candidate was elected, and what was his majority?

7. By the three processes, Numeration, Addition and Subtraction, which have now been taught, all or nearly all arithmetical questions can in theory be solved. But in many cases the operation would be so long as to render the solution an impossibility in practice. The Chapters which follow deal with *Contractions of these processes.*

CHAPTER V.

MULTIPLICATION.

1. Find the amount of money contained in six bags if each holds £7. 8*s.* 9*d.*

£.	*s.*	*d.*
7	8	9
7	8	9
7	8	9
7	8	9
7	8	9
7	8	9
44	12	6

Instead of writing down £7. 8*s.* 9*d.* six times, we indicate this REPETITION thus: £7. 8*s.* 9*d.* $\times$ 6, read £7. 8*s.* 9*d.* *multiplied by* 6. If we know, without actually adding, that six TIMES nine are fifty-four, that six times eight are forty-eight, and that six times seven are forty-two, we can shorten the work thus: £7. 8*s.* 9*d.* $\times$ 6 $=$ £44. 12*s.* 6*d.*

Wording: 6 times 9 pence are 54 pence, put down 6 pence and carry 4 shillings; 6 times 8 shillings are 48 shillings, which, with the 4 shillings carried, make 52 shillings, put down 12 shillings and carry £2; six times £7 are £42, and £2 make £44. Total, £44. 12*s.* 6*d.*

2. Learn by heart: *This sign* ($\times$) *is called* MULTIPLIED BY, *and means that the number of things before it is to be* REPEATED AS MANY TIMES *as is indicated by the number following it.*

The word Multiplication is derived from the Latin *multi-plex*, many-fold.

3. Learn by heart: *The number or quantity which is to be multiplied is called the* MULTIPLICAND, *the number by which we multiply is called the* MULTIPLIER, *and the result is called the* PRODUCT. *The Multiplier and Multiplicand are often also called* FACTORS *of the Product.*

4. $3 \times 4 = 12$; $4 \times 3 = 12$; $\therefore 3 \times 4 = 4 \times 3$. The following demonstration shews that this holds true of any two numbers whatever:

· · · ·
· · · ·
· · · ·

These dots read horizontally yield three fours, and read vertically they yield four threes. Similarly, seven columns of eight dots each would form eight lines of seven dots each, and so on with any other two numbers. Hence we can extend the meaning of the sign × when placed between two abstract numbers.

Learn by heart: *The sign* (×) *indicates that the number on either side of it is to be* REPEATED AS MANY TIMES *as is indicated by the number on the other side of it.*

Even if one of the numbers be concrete, say £7. 8*s.* 9*d.*, it is optional to indicate the multiplication by 6, either by £7. 8*s.* 9*d.* × 6 (£7. 8*s.* 9*d.* *multiplied by* 6), or by 6 × £7. 8*s.* 9*d.* (6 *times* £7. 8*s.* 9*d.*).

5. From what has been said in § 1 of this Chapter, it follows that a certain series of multiplications already performed must be learnt by heart. This series is called the MULTIPLICATION TABLE.

Question. How much is 1 × 2 (once two)?

Answer. 2.

Q. How many twos in 2?

A. One.

Q. One *what?*

A. One two in 2.

Q. How many ones in 2?

A. 2 ones in 2.

Q. What is the half of 2?

A. 1.

Q. Which is more, 2 × 2, or 1 × 2?

A. 2 × 2.

Q. By how much?

A. By 2.

Q. How much, then, is 2 × 2 (twice two)?

A. 4.

Q. How many twos in 4?

A. 2 twos in 4.

Q. How many ones in 4 ?

A. 4 ones in 4.

Q. What is the half of 4 ?

A. 2.

Q. If 4 things are divided equally among 4 persons, what will each person have ?

A. 1 thing.

Q. Therefore what is the quarter of 4 ?

A. 1.

Q. Which is more, 3×2 (3 times 2), or 2×2 ?

A. 3×2.

Q. By how much ?

A. By 2.

Q. Therefore how much is 3×2 ?

A. 6.

Q. Which is more, 2×3, or 3×2 ?

A. Both the same.

Q. Prove it by arranging dots.

A. : : :

Q. How many twos are there in 6 ?

A. 3 twos in 6.

Q. How many threes in 6 ?

A. 2 threes in 6.

Q. What is the half of 6 ?

A. 3.

Q. If 6 things are divided equally among 3 persons, what will each person have ?

A. 2 things.

Q. Therefore what is the third part of 6 ?

A. 2.

Q. What is the sixth part of 6 ?

A. 1.

Q. Which is more, 4×2, or 3×2 ?

A. 4×2.

Q. By how much?

A. By 2.

Q. Therefore how much is 4×2?

A. 8.

Q. Which is more, 4×2, or 2×4?

A. Both the same.

Q. Therefore how many twos in 8?

A. 4 twos in 8.

Q. How many fours in 8?

A. 2 fours in 8.

Q. What is the half of 8?

A. 4.

Q. What is the quarter of 8?

A. 2.

Q. What is the eighth part of 8?

A. 1.

Q. Which is more, 5×2, or 4×2?

A. 5×2.

Q. By how much?

A. By 2.

Q. Therefore how much is 5×2?

A. 10.

Q. Which is more, 5×2, or 2×5?

A. Both the same.

Q. Therefore how many twos in 10?

A. 5 twos in 10.

Q. How many fives in 10?

A. 2 fives in 10.

Q. What is the half of 10?

A. 5.

Q. What is the fifth part of 10?

A. 2.

Q. What is the tenth part of 10?

A. 1.

Q. Which is more, 6 × 2, or 5 × 2?
A. 6 × 2.
Q. By how much?
A. By 2.
Q. How much is 6 × 2?
A. 12.
Q. Which is more, 6 × 2, or 2 × 6?
A. Both the same.
Q. How many twos in 12?
A. 6 twos in 12.
Q. How many sixes in 12?
A. 2 sixes in 12.
Q. Therefore what is the half of 12?
A. 6.
Q. What is the sixth part of 12?
A. 2.
Q. What is the twelfth part of 12?
A. 1.
And so on, up to 10 times 2.

Learn by heart:

1 × 2 = 2 (read, once two is two).	6 × 2 = 12
2 × 2 = 4 (twice two is four).	7 × 2 = 14
3 × 2 = 6 (three times two is six).	8 × 2 = 16
4 × 2 = 8	9 × 2 = 18
5 × 2 = 10	10 × 2 = 20

6. Multiply £487. 9*s*. 10*d*. by 2.

£.	*s.*	*d.*
487	9	10
		2
974	19	8

Wording: 20 (pence), **8′**, carry 1; 18, **19′** (shillings); **14′**, carry 1; 16, **17′**, carry 1; 8, **9′**.

Double £7468. 18*s*. 11¾*d*.

£.	*s.*	*d.*
7468	18	11¾
		2
14937	17	11½

Wording: 6, ½′*d*., carry 1; 22, 23, **11′** (pence), carry 1; 16, **17′** (shillings), carry 1; 2, 3 (half-sovs.), **1′**, carry 1; 16, **17′**, carry 1; 12, **13′**, carry 1; 8, **9′**; **14′**.

Find twice 4,805,639.

4805639	*Wording:* **18′** carry 1; 6, **7′**; **12′**, carry 1; 10,
2	**11′**; **16′**, carry 1; 8, **9′**.
9611278	

Multiply £3,276,003. 17*s.* 10¾*d.* by 2.

£. *s.* *d.*	
3276003 17 10¾	*Wording:* 6, ½′*d.*, carry 1; 20, 21, **9′**, carry 1;
2	14, **15′**, carry 1; 2, 3, **1′**, carry 1; 6, **7′**; **0′**; **0′**;
6552007 15 9½	**12′**, carry 1; 14, **15′**, carry 1; 4, **5′**; **6′**.

EXERCISE XI. (*a*).

Apply this table to the following:

(1) How much money will there be in 2 bags, if each contains £245. 13*s.* 7¼*d.*?

(2) If I travel from London to Liverpool and back, the distance between them being 192 miles, how many miles shall I have travelled?

(3) How many soldiers are there in 2 regiments of 876 soldiers each?

(4) Find the cost of a pair of ponies costing £15. 17*s.* 6*d.* each.

(5) A travels 487 miles north, and B the same distance south. How far will they be apart?

(6) Find the double of £2493. 15*s.* 9¾*d.*

(7) 416296 × 2	(14) £34088. 13*s.* 9½*d.* × 2
(8) £40529. 6*s.* 0½*d.* × 2	(15) 639756 × 2
(9) 278409 × 2	(16) £62977. 5*s.* 3¼*d.* × 2
(10) £51732. 17*s.* 2¾*d.* × 2	(17) 847538 × 2
(11) 301620 × 2	(18) £79863. 19*s.* 10*d.* × 2
(12) £23154. 8*s.* 8¾*d.* × 2	(19) 925317 × 2
(13) 510847 × 2	(20) £18641. 7*s.* 11¼*d.* × 2

7. Treat 1 × 3, 2 × 3, 3 × 3, and so on, up to 10 × 3, as the table of Twos has been treated in § 5; but in examining products, such as 4 × 3, introduce also the recapitulatory questions, "What is the half of 12?" and "What is the sixth part of 12?" Teach in the same way each portion of the Multiplication Table up to 10 × 10. It will be found that the effort and time have been most profitably spent, and will save much future labour.

Learn by heart:

$1 \times 3 = 3$	$6 \times 3 = 18$
$2 \times 3 = 6$	$7 \times 3 = 21$
$3 \times 3 = 9$	$8 \times 3 = 24$
$4 \times 3 = 12$	$9 \times 3 = 27$
$5 \times 3 = 15$	$10 \times 3 = 30$

Point out that the sum of the digits in the successive answers follows the order 3, 6, 9.

Exercise XI. (*b*).

(1) How much money will there be in 3 bags, if each contains £567. 18*s*. $11\frac{1}{4}$*d*.?

(2) Find the length of the three sides of a triangle, if each side is 493 feet long?

(3) How much do I require to give £2045. 7*s*. 6*d*. to each of 3 persons?

(4) How many soldiers in 3 armies of 389,012 men each?

(5) Find the value of 3 East-Indiamen, if each is worth £645,900.

(6) What was a man's fortune if he left £66,666. 13*s*. 4*d*. to each of his 3 children?

(7) 416296×3

(8) £40529. 6*s*. $0\frac{1}{2}$*d*. $\times 3$

(9) 278409×3

(10) £51732. 17*s*. $2\frac{3}{4}$*d*. $\times 3$

(11) 301620×3

(12) £23154. 8*s*. $8\frac{3}{4}$*d*. $\times 3$

(13) 510847×3

(14) £34088. 13*s*. $9\frac{1}{2}$*d*. $\times 3$

(15) 639756×3

(16) £62977. 5*s*. $3\frac{1}{4}$*d*. $\times 3$

(17) 847538×3

(18) £79863. 19*s*. 10*d*. $\times 3$

(19) 925317×3

(20) £18641. 7*s*. $11\frac{1}{4}$*d*. $\times 3$

8. Learn by heart:

$1 \times 4 = 4$	$6 \times 4 = 24$
$2 \times 4 = 8$	$7 \times 4 = 28$
$3 \times 4 = 12$	$8 \times 4 = 32$
$4 \times 4 = 16$	$9 \times 4 = 36$
$5 \times 4 = 20$	$10 \times 4 = 40$

Exercise XI. (*c*).

(1) How much money in 4 bags, if each contains £765. 0*s*. 11*d*.?

(2) Find the length of the sides of a square, if each is 894 feet long.

(3) How much do I owe altogether, if to each of 4 men I owe £321. 19*s.* 10*d.* ?

(4) How many students in 4 colleges, if there are 465 in each?

(5) 1 lb. Troy has 5760 grains. How many grains in 4 lbs.?

(6) 1 cwt. (hundredweight) has 4 qrs. (quarters), and each qr. has 28 lbs. How many lbs. in a cwt.?

(7) 416296 × 4
(8) £40529. 6*s.* 0½*d.* × 4
(9) 278409 × 4
(10) £51732. 17*s.* 2¾*d.* × 4
(11) 301620 × 4
(12) £23154. 8*s.* 8¾*d.* × 4
(13) 510847 × 4
(14) £34088. 13*s.* 9½*d.* × 4
(15) 639756 × 4
(16) £62977. 5*s.* 3¼*d.* × 4
(17) 847538 × 4
(18) £79863. 19*s.* 10*d.* × 4
(19) 925317 × 4
(20) £18641. 7*s.* 11¼*d.* × 4

9. Learn by heart:

$1 \times 5 = 5$
$2 \times 5 = 10$
$3 \times 5 = 15$
$4 \times 5 = 20$
$5 \times 5 = 25$
$6 \times 5 = 30$
$7 \times 5 = 35$
$8 \times 5 = 40$
$9 \times 5 = 45$
$10 \times 5 = 50$

Point out that the figure in the units' place is always either 5 or 0.

EXERCISE XI. (*d*).

(1) How much money in 5 bags, if each contains £4039. 18*s.* 7¾*d.*?

(2) How many petals are there in 376 forget-me-nots?

(3) What would be my income in 5 years at £2765. 10*s.* 10*d.* a-year?

(4) Find the length of the sides of a pentagon, if each is 137 inches long.

(5) A spends 5 times as much as B, whose yearly outlay is £941. 7*s.* 4*d.* Find A's expenditure.

(6) How far will a wheel 5 yards in circumference travel in making 3068 turns?

(7) 416296 × 5
(8) £40529. 6*s.* 0½*d.* × 5
(9) 278409 × 5
(10) £51732. 17*s.* 2¾*d.* × 5
(11) 301620 × 5
(12) £23154. 8*s.* 8¾*d.* × 5
(13) 510847 × 5
(14) £34088. 13*s.* 9½*d.* × 5
(15) 639756 × 5
(16) £62977. 5*s.* 3¼*d.* × 5
(17) 847538 × 5
(18) £79863. 19*s.* 10*d.* × 5
(19) 925317 × 5
(20) £18641. 7*s.* 11¼*d.* × 5

10. Learn by heart:

$1 \times 6 = 6$	$6 \times 6 = 36$
$2 \times 6 = 12$	$7 \times 6 = 42$
$3 \times 6 = 18$	$8 \times 6 = 48$
$4 \times 6 = 24$	$9 \times 6 = 54$
$5 \times 6 = 30$	$10 \times 6 = 60$

Point out that the sum of the digits in the successive answers follows the order 6, 3, 9.

Exercise XI. (*e*).

(1) How much money in 6 bags of £4807. 6*s.* 11¼*d.* each?

(2) Find the number of pages in 6 volumes, if each volume has 483 pages.

(3) Find the cost of 6 locomotives, if each costs £2095. 13*s.* 4*d.*

(4) Find the length of the sides of a hexagon, if each is 529 inches in length.

(5) The daily expenditure of an office in the city is £17. 13*s.* 10*d.* How much is this a-week?

(6) A mile has 880 fathoms. How many feet has it (a fathom being 6 feet)?

(7) 416296×6	(14) £34088. 13*s.* 9½*d.* × 6
(8) £40529. 6*s.* 0½*d.* × 6	(15) 639756×6
(9) 278409×6	(16) £62977. 5*s.* 3¼*d.* × 6
(10) £51732. 17*s.* 2¾*d.* × 6	(17) 847538×6
(11) 301620×6	(18) £79863. 19*s.* 10*d.* × 6
(12) £23154. 8*s.* 8¾*d.* × 6	(19) 925317×6
(13) 510847×6	(20) £18641. 7*s.* 11¼*d.* × 6

11. Learn by heart:

$1 \times 7 = 7$	$6 \times 7 = 42$
$2 \times 7 = 14$	$7 \times 7 = 49$
$3 \times 7 = 21$	$8 \times 7 = 56$
$4 \times 7 = 28$	$9 \times 7 = 63$
$5 \times 7 = 35$	$10 \times 7 = 70$

Exercise XI. (*f*).

(1) How much money in 7 bags, each containing £6429. 15*s.* 3½*d.*?

(2) How many minutes in a week, a day consisting of 1440 minutes?

(3) If living costs me 13*s.* 9½*d.* a-day, how much do I require a-week?

(4) Find the length of the sides of a heptagon, each side being 748 inches long.

(5) Find the difference between 52 weeks and a year (of 365 days).

(6) The seventh part of a ton is 320 lbs. How many lbs. are there in a ton?

(7) 416296 × 7
(8) £40529. 6*s.* 0½*d.* × 7
(9) 278409 × 7
(10) £51732. 17*s.* 2¾*d.* × 7
(11) 301620 × 7
(12) £23154. 8*s.* 8¾*d.* × 7
(13) 510847 × 7
(14) £34088. 13*s.* 9½*d.* × 7
(15) 639756 × 7
(16) £62977. 5*s.* 3¼*d.* × 7
(17) 847538 × 7
(18) £79863. 19*s.* 10*d.* × 7
(19) 925317 × 7
(20) £18641. 7*s.* 11¼*d.* × 7

12. Learn by heart:

$1 \times 8 = 8$
$2 \times 8 = 16$
$3 \times 8 = 24$
$4 \times 8 = 32$
$5 \times 8 = 40$
$6 \times 8 = 48$
$7 \times 8 = 56$
$8 \times 8 = 64$
$9 \times 8 = 72$
$10 \times 8 = 80$

Exercise XI. (*g*).

(1) How much money in 8 bags, each containing £5786. 13*s.* 7½*d.*?

(2) A mile has 8 furlongs, and a furlong has 660 feet. How many feet in a mile?

(3) If one man pays 3*s.* 9¼*d.* for his dinner, what will a party of 8 pay?

(4) Find the length of the sides of an octagon, if each side is 237 inches long.

(5) Find the cost of 8 railway tickets at £1. 15*s.* 6*d.* each.

(6) A certain city has 47,968 houses; if, on an average, each has 8 windows, how many windows are there altogether?

(7) 416296 × 8
(8) £40529. 6*s.* 0½*d.* × 8
(9) 278409 × 8
(10) £51732. 17*s.* 2¾*d.* × 8
(11) 301620 × 8
(12) £23154. 8*s.* 8¾*d.* × 8
(13) 510847 × 8
(14) £34088. 13*s.* 9½*d.* × 8
(15) 639756 × 8
(16) £62977. 5*s.* 3¼*d.* × 8
(17) 847538 × 8
(18) £79863. 19*s.* 10*d.* × 8
(19) 925317 × 8
(20) £18641. 7*s.* 11¼*d.* × 8

13. Learn by heart:

$1 \times 9 = 9$	$6 \times 9 = 54$
$2 \times 9 = 18$	$7 \times 9 = 63$
$3 \times 9 = 27$	$8 \times 9 = 72$
$4 \times 9 = 36$	$9 \times 9 = 81$
$5 \times 9 = 45$	$10 \times 9 = 90$

Point out that the sum of the digits is *always* 9; moreover, that the figure in the tens' place is always one less than the multiplier. Thus in 6×9, the tens' figure is 5, and the units' figure the difference between 5 and 9.

EXERCISE XI. (*h*).

(1) How much money in 9 bags, each containing £5437. 6*s.* 7½*d.*?

(2) How many ninepins are there in 403 sets?

(3) What salary shall I draw in 9 months at £13. 2*s.* 6*d.* a-month?

(4) Find the length of the sides of a nonagon, each side being 222 inches long.

(5) What will be the cost of a terrace of 9 houses at £1166. 13*s.* 4*d.* each?

(6) A sovereign weighs 123 grains, how many grains should 9 sovereigns weigh?

(7) 416296 × 9
(8) £40529. 6*s.* 0½*d.* × 9
(9) 278409 × 9
(10) £51732. 17*s.* 2¾*d.* × 9
(11) 301620 × 9
(12) £23154. 8*s.* 8¾*d.* × 9
(13) 510847 × 9
(14) £34088. 13*s.* 9½*d.* × 9
(15) 639756 × 9
(16) £62977. 5*s.* 3¼*d.* × 9
(17) 847538 × 9
(18) £79863. 19*s.* 10*d.* × 9
(19) 925317 × 9
(20) £18641. 7*s.* 11¼*d.* × 9

14. Learn by heart:

$1 \times 10 = 10$	$6 \times 10 = 60$
$2 \times 10 = 20$	$7 \times 10 = 70$
$3 \times 10 = 30$	$8 \times 10 = 80$
$4 \times 10 = 40$	$9 \times 10 = 90$
$5 \times 10 = 50$	$10 \times 10 = 100$

Multiply 68437 by 10.

$$\begin{array}{r} 68437 \\ 10 \\ \hline 684370 \end{array}$$

Wording: 70, 0′, carry 7; 30, 37′, carry 3; 40, 43′, carry 4; 80, 84′, carry 8; 60, 68′.

Similarly 51376 × 10 = 513760, and 123456 × 10 = 1234560. Notice that the figures in the product are the same as those in the multiplicand, with the addition of a cipher in the units' place. The question arises, Will this always be so? On trying any number of cases, the coincidence will be found to hold. If we were to trust to experience only, which is called reasoning by analogy, our conclusions might be correct, but would not carry to our minds the full conviction that is forced upon us by logical deduction. In many cases of reasoning we are obliged to rely upon mere analogy; but in this case we can prove that the coincidence not only will, but must, always hold.

Compare the multiplicand and product in the above example:

XM.	M.	C.	X.	I.		CM.	XM.	M.	C.	X.	I.
6	8	4	3	7	and	6	8	4	3	7	0.

The 7 in the former stood for	7 units,	in the latter for	7 tens.
3	,,	3 tens,	,,	3 hundreds.
4	,,	4 hundreds,	,,	4 thousands.
8	,,	8 thousands,	,,	8 ten-thousands.
6	,,	6 ten-thousands,	,,	6 hundred-thousands.

Thus we see that, by putting the cipher in the units' place, each figure in the multiplicand has been moved one place higher in the numeration scale, and has therefore been made ten times as valuable, or has been *multiplied by* 10, and multiplying each figure by 10 is equivalent to multiplying the whole by 10.

Learn by heart: *To multiply any number by* 10, *put on a cipher in the units' place, and copy the remaining figures.*

Q. Apply this rule to the following: £369. 17*s.* 8$\frac{3}{4}$*d.* × 10.
A. £369. 17*s.* 8$\frac{3}{4}$0*d.*

Teacher. It is evident that the rule as it stands does not apply here, and for this reason: We are dealing with different numeration scales in the same sum, and the cipher as placed above does not alter the positions of the different figures in their respective scales. There being four different scales, four ciphers will be required, thus: £3690. 170*s.* 80$\frac{30}{4}$*d.* This product, though not inaccurate, is evidently inconvenient, and we therefore proceed as follows:

£.	s.	d.
369	17	$8\frac{3}{4}$
		10
3698	17	$3\frac{1}{2}$

Wording: 30, $\frac{1}{2}$′*d.*, carry 7; 80, 87, **3**′, carry 7; 70, 77′, carry 7; 10, 17, 1′, carry 8; **8**′, **9**′, **6**′, **3**′. The pounds need not be worked, and we have only to put the figure carried from the half-sovereigns in the place of the cipher that is added in multiplying the pounds by 10.

EXERCISE XI. (*i*)

(1) How much money in 10 bags, each containing £5846. 17*s.* $10\frac{3}{4}$*d.*?

(2) How many fingers (and thumbs) will 798 men have?

(3) Find the value of 10 shares at £93. 17*s.* 6*d.* each.

(4) How many hundreds in 5786 thousands?

(5) Find the expenditure of 10 years at £1048. 19*s.* 10*d.* a-year.

(6) How many tens in 75964 hundreds?

(7) 416296 × 10	(14) £34088. 13*s.* $9\frac{1}{2}$*d.* × 10
(8) £40529. 6*s.* $0\frac{1}{2}$*d.* × 10	(15) 639756 × 10
(9) 278409 × 10	(16) £62977. 5*s.* $3\frac{1}{4}$*d.* × 10
(10) £51732. 17*s.* $2\frac{3}{4}$*d.* × 10	(17) 847538 × 10
(11) 301620 × 10	(18) £79863. 19*s.* 10*d.* × 10
(12) £23154. 8*s.* $8\frac{3}{4}$*d.* × 10	(19) 925317 × 10
(13) 501847 × 10	(20) £18641. 7*s.* $11\frac{1}{4}$*d.* × 10

15. The interpretation of the symbol ×, even as extended in § 4, is inapplicable to such a case as £5. 3*s.* 10*d.* × 1, since in the symbol 1 the notion of *repetition* does not enter. The interpretation must therefore be still further extended; but every extension of the interpretation of symbols in Arithmetic and throughout Mathematics must fulfil the two following conditions:

(*a*) It must give an intelligible meaning to the symbol in the new case which requires this extension.

(*b*) The new wording must not alter the sense attached to the symbol in the earlier cases, and must remain subject to the general rules already established. For example, whatever extension of meaning we may in future have occasion to give to the symbol +, it must always be true that the sum of two or more numbers will be

the same, in whatever order the addenda be taken; or whatever meaning we may in future have occasion to give to the symbol $\times$, it must always be true that the product of two numbers will be the same in whatever order they are taken.

Learn by heart: *This sign* ($\times$) *indicates that the number on either side of it is to be* TAKEN *as many* TIMES *as is indicated by the number on the other side of it.*

16. Test of accuracy by casting out nines. 32645×8.

$$\begin{array}{r} 32645 \\ 8 \\ \hline 261160 \end{array}$$

Cast out nines from the multiplier and multiplicand and write the results (8 and 2) in the spaces right and left of a cross, as in the margin; multiply these results (16), cast out nines, and place the result (7) in the upper space; lastly, cast out nines from the product found, and write the result in the lower space. If the figures in the upper and lower spaces do not agree, there must be an error in the working.

EXERCISE XII. (*a*).

(1) Find the cost of 7 articles, at £3. 4*s.* 5*d.* each.

(2) Multiply £68419. 11*s.* 2½*d.* by 1.

(3) Multiply £768. 13*s.* 9*d.* by 10.

(4) Multiply 7686875 by 10.

(5) Repeat £9. 13*s.* 8¼*d.* 6 times.

(6) Simplify 9684375 + 9684375 + 9684375 + 9684375.

(7) Take £19. 14*s.* 2¾*d.* 7 times.

(8) Find the amount of 7 collections, each containing 197114583 things.

(9) What shall I spend in 5 years if I spend £476. 13*s.* 8*d.* a-year?

(10) How many men are there in 6 regiments of 936 men each?

(11) How many sheep in 8 flocks of 147 sheep each?

(12) How much must be paid for 6 locomotives at £3126 each?

(13) How far shall I travel in 9 days, if I travel 672 miles a-day?

(14) Find the double of £5943. 14*s.* 6*d.*

(15) Find the length of 4 journeys of 1768 miles each.

(16) The Bank of England sent to the Bank of France 3 chests of money, each containing £147,916. How much money in all?

(17) How many legs have 683 sheep, 429 oxen, and 715 horses?

(18) How many legs have 326 men, 519 ostriches, and 478 canaries?

(19) How many toes have they?

(20) I bought 9 articles at 13*s.* 10½*d.* each; 6 of them I sold at 15*s.* 4½*d.* each, and 3 at £1 each. How much did they all cost me, for how much did I sell them all, and what profit did I make?

EXERCISE XII. (*b*).

(1) To 7 times 4709, add 7037.

(2) To 4 times 6835, add 5 times 216.

(3) Find the difference between 6 times 5987 and 3 times 9412.

(4) Which is the greater, and by how much, 8×3104, or 7×4080?

(5) Multiply the sum of £8417. 13*s.* 11*d.*, £359. 16*s.* 8*d.*, £1043. 3*s.* 2½*d.*, £6. 11*s.* 10¼*d.*, £428. 5*s.* 9½*d.*, and £7932. 17*s.* 11*d.*, by 9.

(6) Multiply the difference between £10,000 and £8342. 17*s.* 10¼*d.* by 10.

(7) Multiply the difference between 6×2304 and 8×1728 by 10.

(8) Multiply the sum of 18451 and 18444 by their difference.

(9) Prove that 1 × £3267. 12*s.* 10*d.* + 2 × £3267. 12*s.* 10*d.* + 3 × £3267. 12*s.* 10*d.* + 4 × £3267. 12*s.* 10*d.* + 5 × £3267. 12*s.* 10*d.* + 6 × £3267. 12*s.* 10*d.* + 7 × £3267. 12*s.* 10*d.* + 8 × £3267. 12*s.* 10*d.* + 9 × £3267. 12*s.* 10*d.* = 9 × £3267. 12*s.* 10*d.* taken 5 times.

(10) Prove that the same is true of 105894.

(11) Also of £1084. 7*s.* 11¼*d.*

(12) Also of 67321.

(13) Also of £4729. 18*s.* 3*d.*

(14) Also of 568139.

(15) Also of £65,478. 17*s.* $9\frac{3}{4}$*d.*

17. Let us compare the meanings of the three symbols +, —, ×, in accordance with the interpretations given in Ch. III. § 2, Ch. IV. § 2, and Ch. V. § 15.

7 chairs + 3 chairs; this means that 7 chairs and 3 chairs are to be added together. *Ans.* 10 chairs.

Again, 7 chairs — 3 chairs; this means that 3 chairs are to be taken away from 7 chairs. *Ans.* 4 chairs.

Again, 7 chairs × 3 chairs; this should mean either that 7 chairs are to be taken 3 chairs' times, or that 3 chairs are to be taken 7 chairs' times; but what sense can we attach to either of these expressions? Evidently none whatever. Let us then investigate a still earlier case.

1 chair + 1 chair? *Ans.* 2 chairs.

1 chair — 1 chair? *Ans.* 0 chairs.

1 chair × 1 chair? This also is perfectly, and perhaps more obviously, meaningless.

Let us therefore consider the first introduction of the symbol × in § 1 of this Chapter. £7 + £7 + £7 + £7 + £7 + £7 has been contracted into £7 × 6, or 6 × £7; similarly, 7 chairs + 7 chairs + 7 chairs + 7 chairs + 7 chairs is contracted into 6 × 7 chairs, or 7 chairs × 6. In every case, then, the 6 has one and the same meaning, viz. 6 *times*, and is perfectly independent of the nature of the thing represented by the 7. Hence such a problem as £7. 8*s.* 10*d.* × £3. 4*s.* $11\frac{1}{4}$*d.* is sheer nonsense, and admits of no solution.*

Learn by heart: *In Addition and Subtraction we must always have the* SAME *kind of units, viz. so many things of one kind added to or taken from so many things of the* SAME *kind; but in Multiplication*

* In Algebra, when the first problem of multiplication presents itself, a still further extension of the meaning of the symbol × becomes necessary; but even then there will be insuperable difficulties in attaching a *definite* meaning to this problem.

we must always have TWO DIFFERENT *kinds of units, viz. so many* THINGS *repeated or taken so many* TIMES.

18. Apply these considerations to the following:

£8 + £0. *Ans.* £8.

£0 + £8. *Ans.* £8.

£0 × 8. *Ans.* £0.

£8 × 0. This *new* case requires investigation. It cannot mean that £8 are not to be multiplied at all, but only left alone, for then it would be equivalent to £8 × 1; the only sense attachable to the expression is that £8 are to be taken *no* times, and therefore that £8 × 0 = £0.*

Or again: We have seen that 7 times 6 = 6 times 7 (§ 4); therefore 0 times 8 should = 8 times 0, as the above reasoning shews to be true, and this strengthens our conviction that we have reasoned correctly.

£8 — £0. *Ans.* £8.

£0 — £8. This has no meaning.

Learn by heart: *In Addition and Multiplication the numbers may be taken in any order we please, but not so in Subtraction.*

CHAPTER VI.

MULTIPLICATION—*continued.*

1. It is required to multiply £587. 13*s.* 11*d.* by 13. As our knowledge of the Multiplication Table only extends to 10 times, some artifice must be adopted by which this defect can be supplied.

* "There is a number of boxes, none of which contain anything. How much do all together contain?

"If a be the number of boxes, then 0 repeated a times, or $a \times 0$, is 0.

"There is a box full of gold, of which no part whatsoever belongs to A. How much belongs to A?

"If p be the number of pounds of gold in the box, then A's part is $0 \times p$, or 0."

(*De Morgan's Algebra*, Introduction.)

Here (×) has the fuller meaning, which we shall give hereafter. See Book II. Ch. III. § 3.

$13 = 10 + 3 = 9 + 4 = 8 + 5 = 7 + 6$, $\therefore$ to repeat any quantity 13 times we may repeat it first 10 and then 3 times, or else first 9 and then 4 times, or 8 and 5 times, or 7 and 6 times, and the sum of each pair of products must be the same.

Thus a person who cannot carry more than 10 boxes of a given weight, can remove 13 boxes from one place to another *in the smallest possible number of journeys*, by dividing his work in any one of the four ways given above. Applying this principle to the problem proposed, we may proceed as follows:

1st mode.

A	£587	13	11
		$13 = 10 + 3$	
B	5876	19	2
C	1763	1	9
D	7640	0	11

The line marked D consists of the sum of the lines B and C, $\therefore$ $D = B + C$; again, B was obtained by taking A 10 times, $\therefore$ $B = 10 \times A$; similarly, $C = 3 \times A$, $\therefore$ $B + C = 10 \times A + 3 \times A$, or $13 \times A$; but $B + C = D$, $\therefore$ $D = 13 \times A$.

2nd mode.

A	£587	13	11
		$13 = 9 + 4$	
B	5289	5	3
C	2350	15	8
D	7640	0	11

$D = B + C = 9 \times A + 4 \times A = 13 \times A$.

3rd mode.

£587	13	11
	$13 = 8 + 5$	
4701	11	4
2938	9	7
7640	0	11

4th mode.

£587	13	11
	$13 = 7 + 6$	
4113	17	5
3526	3	6
7640	0	11

Multiply 87495 by 13 in 4 different ways, shewing that the results coincide.

87495	87495	87495	87495
$13 = 10 + 3$	$13 = 9 + 4$	$13 = 8 + 5$	$13 = 7 + 6$
874950	787455	699960	612465
262485	349980	437475	524970
1137435	1137435	1137435	1137435

Inspection of the above four methods of multiplication of an abstract number by 13, shews that the selection of 10 + 3 is the easiest, because multiplication by 10 involves no labour. This is therefore the method commonly chosen for abstract numbers, and will in practice be found the most convenient in money sums also.

Multiplication by 11 and by 12 may be done in one line, because the table for 11 is learnt by one inspection, and that for 12 has already been, incidentally, committed to memory.

1 × 11 = 11	1 × 12 = 12
2 × 11 = 22	2 × 12 = 24
3 × 11 = 33	3 × 12 = 36
4 × 11 = 44	4 × 12 = 48
5 × 11 = 55	5 × 12 = 60
6 × 11 = 66	6 × 12 = 72
7 × 11 = 87	7 × 12 = 84
8 × 11 = 78	8 × 12 = 96
9 × 11 = 99	9 × 12 = 108
10 × 11 = 110	10 × 12 = 120

EXERCISE XIII.

(1) Find all the possible pairs of numbers (none exceeding 10) whose sum is 14.

(2) Do the same with 15, 16, 17, 18, 19.

(3) Multiply £4279. 13*s.* $8\frac{1}{2}$*d.* by 14, in four different ways, shewing that the results coincide.

(4) Multiply 2756983 by 15, in three different ways.

(5) Multiply £835. 11*s.* $10\frac{3}{4}$*d.* by 16, in three different ways.

(6) Multiply 4102568 by 17, in two different ways.

(7) Multiply £6179. 14*s.* $9\frac{1}{4}$*d.* by 18, in two different ways.

(8) Multiply 3490716 by 19.

(9) Multiply £674. 15*s.* 9*d.* by 11.

(10) Multiply 6747875 by 11.

(11) Multiply £18792. 9*s.* $7\frac{1}{4}$*d.* by 12.

(12) Multiply 1356794 by 12.

2. Multiply 478916 by 20.

A	478916
	20 = 10 + 10
B	4789160
C	4789160
D	9578320

The lines B and C being equal, it is evident that, instead of copying line B, we might have multiplied it by 2 (see Ch. V. § 1), thus :

$$\begin{array}{rl}
478916 & \\
20 & = 10+10 = 10 \times 2 \\
\hline
4789160 & \\
2 & \\
\hline
9578320 &
\end{array}$$

Similarly 478916×30 :

1st mode.

$$\begin{array}{rl}
478916 & \\
30 & = 10+10+10 \\
\hline
4789160 & \\
4789160 & \\
4789160 & \\
\hline
14367480 &
\end{array}$$

2nd mode.

$$\begin{array}{lrl}
\text{A} & 478916 & \\
 & 30 & = 10 \times 3 \\
\hline
\text{B} & 4789160 & \\
 & 3 & \\
\hline
 & 14367480 &
\end{array}$$

478916×13 :

$$\begin{array}{lrl}
\text{A} & 478916 & \\
 & 13 & = 10+3 \\
\hline
\text{B} & 4789160 & \\
\text{C} & 1436748 & \\
\hline
\text{D} & 6225908 &
\end{array}$$

Compare the multiplication by 30 with the multiplication by 13. In the first, the line B, or the *ten* line, is multiplied by 3, and this *at once* yields the answer. In the second, it is not line B, but line A, that is multiplied by 3, and this multiplication only yields us line C, which has yet to be added to line B to give the answer, line D. In other words, 30 times is 3 *times* 10 times; 13 times is 10 times *and* 3 times put together.

The above process of multiplication by 30 may be contracted, as the line B is merely a copy of A with the addition of a 0 in the units' place; we may therefore obtain the result in one line by writing down the 0 and multiplying by 3 at once, thus :

$$\begin{array}{r}
478916 \\
30 \\
\hline
14367480
\end{array}$$

Similarly, we multiply by 40 by writing a 0 in the units' place

and multiplying by 4; and so on for 50, 60, 70, 80, 90, where we write a 0 and multiply by 5, 6, 7, 8, 9, respectively.

In multiplication of money, line B is not merely a copy of line A with the addition of a 0, and this contraction is therefore inapplicable.

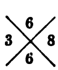
£324. 8*s*. 7*d*. × 50:

£324	8	7
		10
3244	5	10
		5
16221	9	2

Exercise XIV.

(1) £29. 13*s*. 8¼*d*. × 30
(2) £29. 13*s*. 8¼*d*. × 13
(3) 29684375 × 30
(4) 29684375 × 13
(5) £645. 15*s*. 8*d*. × 14
(6) £645. 15*s*. 8*d*. × 40
(7) 6457833 × 40
(8) 6457833 × 14
(9) £10. 16*s*. 9¾*d*. × 50
(10) £10. 16*s*. 9¾*d*. × 15
(11) 10840625 × 50
(12) 10840625 × 15
(13) 9*s*. 10½*d*. × 16
(14) 9*s*. 10½*d*. × 60
(15) 49375 × 16
(16) 49375 × 60
(17) 29364583 × 17
(18) £29. 7*s*. 3½*d*. × 70
(19) £29. 7*s*. 3½*d*. × 17
(20) 29364583 × 70
(21) £7. 18*s*. 5¾*d*. × 18
(22) £7. 18*s*. 5¾*d*. × 80
(23) 79239583 × 18
(24) 79239583 × 80
(25) £56. 19*s*. 11¾*d*. × 19
(26) £56. 19*s*. 11¾*d*. × 90
(27) 569989583 × 19
(28) 569989583 × 90
(29) £58. 17*s*. 8¼*d*. × 15
(30) £58. 17*s*. 8¼*d*. × 50
(31) 58884375 × 15
(32) 58884375 × 50
(33) £194. 17*s*. 2*d*. × 17
(34) £194. 17*s*. 2*d*. × 70
(35) 19485833 × 17
(36) 19485833 × 70
(37) £962. 7*s*. 6*d*. × 19
(38) £962. 7*s*. 6*d*. × 90
(39) 962375 × 19
(40) 962375 × 90

3. Multiply 2628417 by 53.



$$\begin{array}{r} 2628417 \\ 53 = 50+3 \\ \hline 131420850 \\ 7885251 \\ \hline 139306101 \end{array}$$

Multiply £36. 9*s*. 8½*d*. by 67.

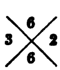
A	£36	9	8½	× 67; 67 = 60 + 7
			10	
B	364	17	1	Line B = 10 × A
			6	„ C = 6 × B = 60 × A
C	2189	2	6	„ D = 7 × A
D	255	7	11½	∴ C + D or E = 60 × A + 7 × A = 67 × A
E	2444	10	5½	

Exercise XV.

(1) £4. 16*s*. 7½*d*. × 23
(2) £4. 16*s*. 7½*d*. × 32
(3) 483125 × 23
(4) 483125 × 32
(5) £12. 13*s*. 5¾*d*. × 34
(6) £12. 13*s*. 5¾*d*. × 43
(7) 126739583 × 34
(8) 126739583 × 43
(9) £26. 17*s*. 11*d*. × 46
(10) £26. 17*s*. 11*d*. × 64
(11) 2689583 × 46
(12) 2689583 × 64
(13) £34. 14*s*. 3¾*d*. × 57
(14) £34. 14*s*. 3¾*d*. × 75
(15) 34715625 × 57
(16) 34715625 × 75
(17) £5. 7*s*. 6¼*d*. × 61
(18) £5. 7*s*. 6¼*d*. × 16
(19) 53760416 × 61
(20) 53760416 × 16
(21) £27. 9*s*. 8¾*d*. × 77
(22) 274864583 × 77
(23) £6. 19*s*. 10¼*d*. × 82
(24) £6. 19*s*. 10¼*d*. × 28
(25) 69927083 × 82
(26) 69927083 × 28
(27) £39. 5*s*. 7½*d*. × 98
(28) £39. 5*s*. 7½*d*. × 89
(29) 3928125 × 98
(30) 3928125 × 89

4. In some cases this process of multiplication may be somewhat contracted, e.g. 56 = 50 + 6, but it also equals 8 + 8 + 8 + 8 + 8 + 8 + 8 = 7 times 8, or 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 8 times 7.

Multiply 78429 by 56.



1st mode.		2nd mode.		3rd mode.	
78429	A	78429	A	78429	A
56 = 50 + 6		56 = 7 × 8		56 = 8 × 7	
3921450	B	627432	E	549003	F
470574	C	7		8	
4392024	D	4392024	D	4392024	D

D = B + C; but B = 50 × A, and C = 6 × A, ∴ B + C = 56 × A.
Again, D = 7 × E; but each E = 8 × A, ∴ D = 7 × 8 × A = 56 × A.
Again, D = 8 × F; but each F = 7 × A, ∴ D = 8 × 7 × A = 56 × A.

Multiply £478. 5*s*. 7*d*. by 56.

1st mode.	2nd mode.	3rd mode.
£478 5 7	£478 5 7	£478 5 7
10	8	7
4782 15 10	3826 4 8	3347 19 1
5	7	8
23913 19 2	26783 12 8	26783 12 8
2869 13 6		
26783 12 8		

Exercise XVI.

(1) Multiply £16. 15*s*. 8¼*d*. by 56 in three different ways, shewing that the results coincide.

(2) 16784375 × 56 in three ways.

(3) £82. 9*s*. 3½*d*. × 45 in three ways.

(4) 82464583 × 45 in three ways.

(5) £143. 6*s*. 10¾*d*. × 20 in four ways.

(6) 15809 × 20 in four ways.

(7) State the different ways in which we can multiply by 24, 30, 36, 42, 60, 63.

5. Multiply 78426 by 100. 100 = 10 × 10, hence we may proceed as follows:

$$\begin{array}{r} 78426 \\ 10 \\ \hline 784260 \\ 10 \\ \hline 7842600 \end{array}$$

But on comparing the product with the multiplicand, we find that we have added two ciphers on the right-hand side. In fact, by the addition of these two ciphers, each digit of the multiplicand has been removed two places higher in the numeration scale, and has thereby had its value raised a hundredfold. Similarly, a number is multiplied by 1000 by adding 3 ciphers on the right, by 10,000 by adding 4 ciphers, and so on.

The numbers 10, 100, 1000, &c., which are 10, 10 × 10, 10 × 10 × 10, &c., are called Powers of 10.

Learn by heart: *To multiply a number by any power of* 10, *put on as many ciphers to the right of the multiplicand as there are ciphers in the multiplier.* This rule is of course inapplicable to multiplication of money. (Ch. V. p. 55.)

Multiply £43. 7*s.* 8½*d.* by 10000.

A	£43	7	8½	
			10	
B	433	17	1	Line B = 10 × A
			10	,, C = 10 × B = 100 × A
C	4338	10	10	,, D = 10 × C = 100 × B = 1000 × A
			10	,, E = 10 × D = 100 × C = 1000 × B = 10000 × A
D	43385	8	4	
			10	
E	433854	3	4	

Exercise XVII.

(1) £529. 17*s.* 8*d.* × 100.
(2) £8342. 5*s.* 9½*d.* × 1000.
(3) £7. 11*s.* 2¾*d.* × 10000.
(4) £845. 4*s.* 10*d.* × 100.
(5) £3410 × 1000000.
(6) 3*s.* 2½*d.* × 100000.
(7) Find the value of a million penny postage stamps.
(8) If one rupee is worth 1*s.* 11¼*d.*, what is the value of a lac of rupees (100,000)?
(9) 42748 × 100.
(10) 609 × 1000.
(11) 5040 × 10000.
(12) 170000 × 100000.
(13) 19 × 1000000.
(14) 1 × 1000.
(15) 3000 × 1000.
(16) 1000 × 100.
(17) 100 × 10000.
(18) 1000 × 1000.
(19) 10000 × 10000.
(20) A million × a million.

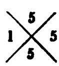
6. Multiply 5267 by 400. $400 = 4 \times 100$; we therefore multiply by 100 and then this product by 4, thus :

5267		
100	or in one line, as in § 2 :	
526700		5267
4		400
2106800		2106800

Multiply £83. 4*s.* $10\frac{1}{4}$*d.* by 600.

£83	4	$10\frac{1}{4}$
		10
832	8	$6\frac{1}{2}$
		10
8324	5	5
		6
49945	12	6

Multiply 78652 by 347. $347 = 300 + 40 + 7$; we therefore take the multiplicand first 300 times, then 40 times, and lastly 7 times, and add these three products together.



78652		
347		
23595600	=	300 times
3146080	=	40 times
550564	=	7 times
27292244	=	347 times

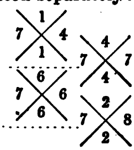
Multiply 43756 by 4768.

43756		
4768		
175024000	=	4000 times
30629200	=	700 times
2625360	=	60 times
350048	=	8 times
208628608	=	4768 times

N.B. The test by casting out nines will not detect an error in the order or position of the figures, since the sum of the digits would be the same whatever their arrangement in the multiplier, the multiplicand or any of the products.

Supposing, however, an error to be indicated by the test, it is not necessary to go through the whole work again, as each line can be tested separately: e.g.

For the 4000 line	the scheme is.........	
,, 700	,,	....
,, 60	,,	....
,, 8	,,	....



Multiply 75948 by 4060.

75948		
4060		
303792000	=	4000 times
4556880	=	60 times
308348880	=	4060 times

13467 × 10110.

13467		
10110		
134670000	=	10000 times
1346700	=	100 times
134670	=	10 times
136151370	=	10110 times

Multiply £15. 7*s.* 8¾*d.* by 347.

	£	s.	d.	
A	£15	7	8¾ × 7	
			10	
B	153	17	3½ × 4	
			10	
C	1538	12	11	
			3	
	4615	18	9	$=3\times C=3\times 100$ times $=300$ times
	615	9	2	$=4\times B=4\times\ 10$ times $=\ 40$ times
	107	14	1¼	$=7\times A=7\times\ 1$ time $=\ 7$ times
	5339	2	0¼	347 times

£6. 15*s.* 10½*d.* × 8040.

	£	s.	d.	
A	£6	15	10½	
			10	
B	67	18	9 × 4	
			10	
C	679	7	6	
			10	
D	6793	15	0	
			8	
	54350	0	0	$=8\times D=8\times 1000$ times $=8000$ times
	271	15	0	$=4\times B=4\times\ 10$ times $=\ 40$ times
	54621	15	0	8040 times

Multiply £3. 7*s.* 8½*d.* by 2400. $2400 = 2000 + 400 = 3 \times 8 \times 100 = 4 \times 6 \times 100 = 2 \times 12 \times 100 = 8 \times 3 \times 100 = 8 \times 100 \times 3 = 3 \times 100 \times 8$, &c. &c.

1st mode.

3	7	8½	
		10	
33	17	1	
		10	
338	10	10 × 4	
		10	
3385	8	4	
		2	
6770	16	8	= 2000 times
1354	3	4	= 400 times
8125	0	0	= 2400 times

2nd mode.

3	7	8½	
		10	
33	17	1	
		10	
338	10	10	= 100 times
		8	
2708	6	8	= 800 times
		3	
8125	0	0	= 3 × 800 times = 2400 times

3rd mode.

£3	7	8½.	
		8	
27	1	8	= 8 times
		3	
81	5	0	= 3 × 8 times = 24 times
		10	
812	10	0	= 10 × 24 times = 240 times
		10	
8125	0	0	= 10 × 240 times = 2400 times

It will be seen that the multiplication by factors (2nd and 3rd modes) requires the fewest figures; it is therefore the more elegant, and, in addition to this, enables us, by altering the order of the factors, to apply an almost absolute verification.

7. Find the cost of 42 articles if 7 cost £7. 8*s.* 10*d.* $42 = 6 \times 7$, ∴ 42 articles cost 6 times as much as 7 cost = 6 × £7. 8*s.* 10*d.*

£7	8	10
		6
44	13	0

If I walk 76 miles in 4 days, how far shall I walk in 20 days? $20 = 5 \times 4$, ∴ in 20 days I shall walk 5 times as far as in 4 days, *that is* 5×76 miles = 380 miles.

8. Simplify $8 \times (16 + 5 - 3)$. $8 \times (16 + 5 - 3)$ means 8 times the quantity within the brackets, therefore we must first simplify $16 + 5 - 3$, which becomes $21 - 3$, or 18. Then 8×18, or 144, is the answer.

9. In multiplying two abstract numbers, it is better to choose as the multiplier that number which has the fewer significant figures.

Exercise XVIII.

(1) £47. 19*s.* $8\frac{3}{4}$*d.* × 371
(2) £612. 9*s.* $7\frac{1}{4}$*d.* × 802
(3) £8. 3*s.* $5\frac{1}{2}$*d.* × 5049
(4) £9. 0*s.* 11*d.* × 62
(5) £50. 17*s.* $10\frac{1}{2}$*d.* × 10010
(6) £471. 13*s.* $9\frac{3}{4}$*d.* × 7021

In two ways.
(7) £25. 7*s.* $4\frac{1}{4}$*d.* × 280
(8) £83. 15*s.* 3*d.* × 6400
(9) £90. 11*s.* $2\frac{1}{2}$*d.* × 180
(10) £308. 19*s.* $1\frac{1}{4}$*d.* × 360
(11) £621. 14*s.* $9\frac{3}{4}$*d.* × 42
(12) £495. 6*s.* 6*d.* × 840

(13) 36724 × 37
(14) 5809 × 1209
(15) 8067 × 2109
(16) 53244 × 5005
(17) 178 × 16359
(18) 50070 × 829
(19) 69532 × 10101
(20) 69532 × 11001
(21) 69532 × 1010
(22) 69532 × 11111
(23) 69532 × 100100
(24) 69532 × 11011
(25) 26418 × 210
(26) 57390 × 4500
(27) 256080 × 120
(28) 7439 × 320
(29) 6428000 × 7200
(30) 4286 × 5500

(31) Find the daily wages of 367 men at 3*s.* $4\frac{1}{2}$*d.* per day each.

(32) How much would this amount to in a year, leaving out 52 Sundays?

(33) Find the cost of 815 articles at 7*s.* $10\frac{3}{4}$*d.* each.

(34) Find the value of 3000 Venetian ducats at 9*s.* 5*d.* each.

(35) Find the value of 94375 Prussian thalers at 2*s.* $10\frac{3}{4}$*d.* each.

(36) Find the value of 1722 railway tickets at 1*s.* 7*d.* each.

(37) Find the value of a cwt. (112 lbs.) of sugar at $4\frac{3}{4}$*d.* per lb.

(38) Find the cost of a ton (20 cwt.) of iron nails at $\frac{3}{4}$*d.* per lb.

(39) What is the charge for translating 74 folios at 2*s.* $4\frac{1}{2}$*d.* each?

(40) Find the cost of a lb. Troy (5760 grains) at $1\frac{3}{4}$*d.* per grain.

(41) What is the yearly rent of a cottage at 3*s.* 3*d.* a-week?

(42) Find the yearly rent of a terrace of 165 houses, of which each pays £136. 10*s.*

(43) Find the strength of an army consisting of 113 regiments of 947 men each.

(44) How many words are there on a page of 29 lines, each line containing 14 words?

(45) How many bricks are there in 306 yards of wall, each requiring 288 bricks?

(46) At the latitude of London, 1 degree (1^0) of longitude is very nearly 37 geographical miles. Find the length of the whole parallel (360^0).

(47) How many eggs are there in 3080 boxes, each containing 4769 eggs?

(48) How many grains in 68340 lbs. Avoirdupois (7000 grains each)?

(49) How many hours in a year of 365 days?

(50) How many inches will a wheel have travelled over in making 5317 turns, if the circumference of the wheel is 127 inches?

(51) How many yards in the equator, which is 24899 miles in length (1 mile = 1760 yards)?

(52) How many feet in the same (1 yard = 3 feet)?

(53) How many inches in the same (1 foot = 12 inches)?

(54) What is the issue of a newspaper in 13 weeks, the daily issue of which is 97428 copies?

(55) The Penny Cyclopædia consists of 27 vols., each vol. has, on an average, 512 pages, each page has two columns, and each column has 81 lines of about 8 words each. How many words are there in all?

(56) How many ounces in 1 ton Avoirdupois? (see Table.)

(57) How many inches in a mile?

(58) How many farthings in £1?

(59) How many pounds in 143 iron plates, each weighing 5 tons?

(60) Sound travels through air at the rate of 1130 feet per second. How many feet distant is a thunder-cloud if the report is heard 17 seconds after the flash is seen?

(61) Shew that $27043 \times 233 + 27043 \times 419 + 27043 \times 326 = 27043 \times 326 \times 3$.

(62) Shew that £7. 11*s.* 10*d.* × 233 + £7. 11*s.* 10*d.* × 419 + £7. 11*s.* 10*d.* × 326 = £7. 11*s.* 10*d.* × 326 × 3.

(63) Find the total cost of 43 articles at 5*s.* 11¼*d.* each, 107 articles at 13*s.* 8½*d.* each, 2160 articles at 3*s.* 9¾*d.* each, and 11 articles at £2. 4*s.* 7*d.* each.

(64) Find the total cost of 243 pieces of T cloths of 30 yards each, at 3¾*d.* per yard; 67 pieces of sheeting of 38 yards each, at 1*s.* 10¼*d.* per yard, 375 pieces of shirtings at 6*s.* 2*d.* per piece, and 18750 yards of fents at 1¼*d.* per yard.

(65) How much change shall I receive out of £100 after paying the following bill: 5 cwt. of sugar at 3½*d.* per lb., a chest of tea containing 53 lbs. at 2*s.* 10½*d.* per lb., half a ton of Carolina rice at 3¼*d.* per lb., 2086 lbs. of raw Ceylon coffee at 5½*d.* per lb?

(66) A farmer sold 437 sheep at £1. 17*s.* 9*d.* each, and bought 23 bullocks at £12. 12*s.* each, and 19 calves at £7. 17*s.* 6*d.* each. How much has he left?

(67) If I spend 1*s.* 9*d.* a-day for a return ticket, travelling six days in the week, how much shall I save in a year (52 weeks) by taking an annual ticket for £25?

(68) I bought 843 articles at £1. 3*s.* 4½*d.* each, and sold them for £1000. What was my profit?

(69) I bought 756 articles for £195, and sold them at 5*s.* 7½*d.* each. What was my gain?

(70) I bought 400 dozens of wine at £1. 15*s.* 6*d.* per dozen, and retailed them at 3*s.* 6*d.* per bottle. Find my profit per dozen; also, in two ways, the profit on the whole.

(71) I bought 325 dozen of wine at 3*s.* 2*d.* per bottle. For how much must the whole be sold to gain £22. 10*s.*?

(72) A trader took out to China 24,178 pieces of grey shirting, of 29 yards each, at 2¼*d.* per yard; 12,089 pieces of print, of 58 yards each, at 4½*d.* per yard; 100,000 yards of flannel at 11½*d.* per yard, and 2400 dozen handkerchiefs at 1*s.* 7½*d.* per dozen. He brought back 3000 chests of tea, of 35 lbs. each, at 10*d.* per lb.; 16,498 lbs. of raw silk at £1. 3*s.* 10½*d.* per lb., and the remainder in cash. How much money did he bring back?

(73) If I pay £13. 13*s.* per quarter for rent, £2. 8*s.* 6*d.* per quarter for rates, £2. 18*s.* 9*d.* a-week for food, 11*s.* 3*d.* a-week for washing, £42 every half-year for school bills, £15. 12*s.* 6*d.* every half-year for life insurance, £35 a-year for clothes, £3. 10*s.* for coals for the summer half and £7. 7*s.* for the winter half-year, and wish to lay by £120 out of an income of £600 a-year, how much shall I have for other expenses?

(74) A publisher sells a certain book at 3*s.* 2*d.* per copy nett; of this, he pays to the printer, $9\frac{1}{4}d.$ per copy; to the binder, $6\frac{3}{4}d.$ per copy; to the author, a royalty of 9*d.* for every copy he sells. Of an edition of 1000 copies, he sells 853; the remainder are left on hand. Will he have lost or gained, and how much?

(75) I bought 2 tons of sugar at $3\frac{1}{4}d.$ per lb.; 7 cwt. 21 lbs. got damaged. I sold the good sugar at 4*d.* a pound, and the damaged sugar at $2\frac{1}{2}d.$ per pound. What was my gain or loss?

(76) A person mixed 23 gallons of Jamaica rum at 9*s.* 7*d.* per gallon, with 18 gallons of British rum at 7*s.* 5*d.* per gallon, and 30 gallons of water; he sold the mixture at 11*s.* 6*d.* per gallon. What was his total gain?

(77) A person bought 427 yards of cloth at 3*s.* 8*d.* per yard; for how much must he sell the whole so as to gain $7\frac{1}{2}d.$ per yard?

(78) If 8 articles cost £4. 7*s.* $10\frac{1}{4}d.$, what will 56 articles cost?

(79) If 9 men can dig 43 yards of trench in a given time, how much will 45 men dig in the same time?

(80) How much in double the time?

(81) If 3 men build a given wall in 14 days, how long would 1 man take to build it?

(82) If 20 men build a given wall in 8 days, how long will 10 men take to do it?

(83) And how long would 4 men take?

(84) If 3 articles cost 11*s.* $8\frac{1}{2}d.$, what will 27 articles cost?

(85) If 8 chairs cost £2. 2*s.* 6*d.*, what will 64 chairs cost?

(86) If one dozen pens cost $3\frac{1}{4}d.$, what will the gross cost?

(87) A man bought 28 Bandana handkerchiefs at 7 for £1. 12*s.* $7\frac{1}{2}d.$; he sold them at 5*s.* 9*d.* each. Find his total profit.

(88) By mistake he entered this profit in his books among the losses; at the end of the month his books shewed a profit of only £83. 10*s*. What ought they to have shewn?

(89) In a given time 9 men can put up 67 yards of fence. How many yards can 63 men put up in treble the time?

(90) If 17 men can dig a given quantity of trench in a given time, how many men will be wanted to dig 4 times the quantity in a quarter of the time?

(91) If out of a sack of flour we make 43 shilling loaves, how many threepenny loaves could we have made of it?

(92) Simplify $17 \times (184 - 47 + 62)$.

(93) Simplify $302 \times (5 \times 17 - 3 \times 11)$; $597 \times (5 \times 17 - 9 \times 9)$.

(94) Simplify $6 \times 7 \times (11 \times 156 + 13 \times 84)$.

(95) Simplify $(81 + 317) \times (24 + 11)$.

(96) Simplify $6 \times 8 \times 9 \times 5 \times 4 \times 317$.

(97) Find the continued product of the first 9 numbers.

(98) Shew that $740625 \times 386 + 740625 \times 234 + 740625 \times 1671 + 740625 \times 176 + 740625 \times 809 + 740625 \times 546 = 740625 \times 546 \times 7$.

(99) Shew that £7. 8*s*. $1\frac{1}{2}d$. $\times 386$ + £7. 8*s*. $1\frac{1}{2}d$. $\times 234$ + £7. 8*s*. $1\frac{1}{2}d$. $\times 1671$ + £7. 8*s*. $1\frac{1}{2}d$. $\times 176$ + £7. 8*s*. $1\frac{1}{2}d$. $\times 809$ + £7. 8*s*. $1\frac{1}{2}d$. $\times 546$ = £7. 8*s*. $1\frac{1}{2}d$. $\times 546 \times 7$.

(100) A bankrupt owes £8745, and can pay 9*s*. $5\frac{3}{4}d$. in the £1. What are his assets?

(101) A railway company has 16,200 shares. Their expenses are £14,175. What must be the company's gross income to yield a dividend of 17*s*. 6*d*. per share?

(102) Multiply the sum of 438619 and 30405 by 198.

(103) Multiply the difference between 438619 and 30405 by 1098.

(104) Multiply the sum of 2815 and 365 by the difference between these two numbers.

(105) From 18 times the product of 519 and 98 take 7 times this product.

CHAPTER VII.

DIVISION.

1. How many *times* can I pay £4. 7*s.* 9½*d.* out of £26. 6*s.* 9*d.*?

£26	6	9	
4	7	9½	1 time
21	18	11½	
4	7	9½	1 ,,
17	11	2	
4	7	9½	1 ,,
13	3	4½	
4	7	9½	1 ,,
8	15	7	
4	7	9½	1 ,,
4	7	9½	
4	7	9½	1 ,,
—	—	—	6 times exactly.

How many times are £5. 8*s.* 10*d.* contained in £43. 17*s.* 11*d.*?

£43	17	11	
5	8	10	1 time
38	9	1	
5	8	10	1 ,,
33	0	3	
5	8	10	1 ,,
27	11	5	
5	8	10	1 ,,
22	2	7	
5	8	10	1 ,,
16	13	9	
5	8	10	1 ,,
11	4	11	
5	8	10	1
5	16	1	
5	8	10	1
0	7	3	8 times and 7*s.* 3*d.* over.

How many times are 112 lbs. contained in 800 lbs.?

800	
112	1 time
688	
112	1 ,,
576	
112	1 ,,
464	
112	1 ,,
352	
112	1 ,,
240	
112	1 ,,
128	
112	1 ,,
16	7 times and 16 lbs. over.

How many men will it require to dig 41 yards of trench in a given time if 1 man can dig 6 yards in that time?

41	
6	1 man
35	
6	1 ,,
29	
6	1 ,,
23	
6	1 ,,
17	
6	1 ,,
11	
6	1 ,,
5	6 men and 5 yards over.

Six men would in the given time leave 5 yards to be done, seven men would be able to do 1 yard more than required; we may choose either answer, but 7 men is the more correct. *Ans.* 7 men and 1 yard to spare.

The process by which we find the *number of times* that one quantity is contained in another is called Division, for which the symbol is ÷.

2. Learn by heart: *This sign* (÷) *is called* DIVIDED BY, *and has the following meaning: Find* HOW MANY TIMES *the quantity following the sign is contained in the quantity preceding the sign, and the answer will be* SO MANY TIMES.

3. Learn by heart: *In Division the quantity to be divided is called the* DIVIDEND, *that by which we divide is called the* DIVISOR, *and the answer is called the* QUOTIENT. *If anything is over it is called the* REMAINDER.

EXERCISE XIX.

(1) How many times are £5. 7*s.* 11*d.* contained in £16. 3*s.* 9*d.*?

(2) £13. 13*s.* 10*d.* ÷ £3. 8*s.* 5½*d.*

(3) If 1 article costs 12*s.* 7½*d.*, how many can I buy for £4. 8*s.* 4½*d.*?

(4) If I save £1. 4*s.* 10½*d.* a-week, how long shall I be in accumulating £9. 19*s.*?

(5) By what number must I multiply £8. 4*s.* 6*d.* to make £41. 2*s.* 6*d.*?

(6) To how many persons can I give £3. 15*s.* each out of £37. 10*s.*?

(7) If I can travel 65 miles a-day, how long will it take me to get over 520 miles?

(8) If one strip of carpet is 115 inches in length, how many strips can I cut from 1035 inches of carpet?

(9) How many regiments of 875 men each are there in 9625 men?

(10) How many yards of wall can I build with 448 bricks if each yard requires 64 bricks?

(11) How many times are £3. 8*s.* 10*d.* contained in £15?

(12) £37. 9*s.* 4*d.* ÷ £4. 13*s.* 7*d.*

(13) If 1 article costs £1. 5*s.* 9*d.*, how many can I buy for £8. 1*s.* 1*d.*?

(14) If I save £2. 12*s.* 6*d.* a month, how long will it take me to accumulate £25?

(15) Out of £18. 4*s.* 9*d.* I bought as many sheep as I could at £1. 15*s.* 6*d.* each, and with the remainder I bought a lamb. Find the cost of the lamb.

(16) Out of £13. 13*s.* I bought as many articles as I could at £1. 10*s.* each. How much more money shall I want to buy one more article?

(17) How many months of 28 days each are there in a year?

(18) If I start Oct. 4th and travel 85 miles a-day, on what day shall I reach a place 450 miles off?

(19) How many periods of 12 minutes are there in 1 hour?

(20) To how many persons can I give £68. 14*s.* 10*d.* if I have £600?

4. If 1 article costs 4*s.* 7*d.*, how many can I buy for £28,450? On trial it would be found that the process employed above would be so long as to be practically useless. Some artifice for contracting the operation is therefore indispensable. (See Ch. IV. § 7.) Make a table as follows:

1 article costs	£0	4	7
10 articles cost	2	5	10
100 ,, ...	22	18	4
1000 ,, ...	229	3	4
10000 ,, ...	2291	13	4
100000 ,, ...	22916	13	4

By means of this table we see that we can buy 100,000 articles, and shall yet have some money remaining.

	£28450	0	0
	22916	13	4
1st remainder	5533	6	8

The table further shews us that with this remainder we can pay not only for 10,000, but for twice 10,000, or 20,000 articles. 20,000 articles cost 2 × £2291. 13*s.* 4*d.* = £4583. 6*s.* 8*d.*

	£5533	6	8	= 1st remainder
	4583	6	8	= cost of 20,000 articles
2nd remainder	950	0	0	

With this second remainder we cannot buy another 10,000, but we can buy much more than 1000 articles, and the question arises:

How many thousands? 1000 articles cost, by the table, £229. 3*s.* 4*d.* To make a rough guess as to the number of times this is contained in the second remainder, £950, we should examine the figures of highest value in each, which are respectively 2 hundreds and 9 hundreds; this leads us to suppose that we can pay for 4 thousands. On trial we find that 4 × £229. 3*s.* 4*d.* = £916. 13*s.* 4*d.*

	£	s.	d.	
	£950	0	0	= 2nd remainder
	916	13	4	= cost of 4000 articles
3rd remainder	33	6	8	

With this third remainder we can pay for 100 articles.

	£	s.	d.	
	£33	6	8	= 3rd remainder
	22	18	4	= cost of 100 articles
4th remainder	10	8	4	

With this fourth remainder we can buy 4 × 10 articles. 4 × £2. 5*s.* 10*d.* = £9. 3*s.* 4*d.*

	£	s.	d.	
	£10	8	4	= 4th remainder
	9	3	4	= cost of 40 articles
5th remainder	1	5	0	

With this fifth remainder we can buy 5 articles. 5 × 4*s.* 7*d.* = £1. 2*s.* 11*d.*

	£	s.	d.	
	£1	5	0	= 5th remainder
	1	2	11	= cost of 5 articles
6th remainder	0	2	1	

Consequently we can pay for 124,145 articles, and shall have 2*s.* 1*d.* over, which will not pay for another article.

Mod. op.:

Table.

£	s.	d.	
	4	7	I.
2	5	10	X.
22	18	4	C.
229	3	4	M.
2291	13	4	XM.
22916	13	4	CM.

	£	s.	d.	
4*s.* 7*d.*)	£28450	0	0	
	22916	13	4	100000 articles
	5533	6	8	
	4583	6	8	20000 ,,
	950	0	0	
	916	13	4	4000 ,,
	33	6	8	
	22	18	4	100 ,,
	10	8	4	
	9	3	4	40 ,,
	1	5	0	
	1	2	11	5 ,,
Remainder......	0	2	1	124145 articles

Answer. 124145 articles, and 2*s.* 1*d.* over.

EXERCISE XX.

(1) How many times are £5. 18*s.* 7*d.* contained in £800. 8*s.* 9*d.*?

(2) How many articles can I buy for £2118. 18*s.* 3½*d.* if each costs £9. 15*s.* 3½*d.*?

(3) By what number must I multiply £15. 13*s.* 7*d.* to obtain £4280. 8*s.* 3*d.* for product?

(4) £4973. 11*s.* 2*d.* ÷ £30. 17*s.* 10*d.*

(5) If I put by £91. 7*s.* a-year, how long shall I take to accumulate £1370. 5*s.*?

(6) I invest £9287. 10*s.* in railway shares, each costing £92. 17*s.* 6*d.* and yielding a yearly income of £3. 10*s.* each. Find my total yearly revenue.

(7) How many Napoleons at 15*s.* 9*d.* each can I get for 189 Prussian thalers at 2*s.* 10*d.* each?

(8) (227 × £16. 11*s.* 4*d.*) ÷ (28 × 18*s.* 11*d.*).

(9) How many guineas are there in £510. 6*s.*?

(10) A earns £9. 2*s.* 6*d.* a-week, and spends £7. 5*s.* a-week. How long will he be in saving £150?

(11) A man's wages are £1. 17*s.* 6*d.* a-week; his wife earns 18*s.* a-week; his 2 sons earn 6*s.* 9*d.* a-week each. How long must the wages of the family remain unpaid to amount to £79. 7*s.*?

(12) How many sovereigns, half-sovereigns, crowns, half-crowns, florins, shillings, sixpences, fourpenny-pieces, threepenny-pieces, pennies, halfpennies, and farthings, an equal number of each, can be got from £358. 17*s.* 5*d.*?

(13) How many times are £3. 18*s.* 10*d.* contained in £429. 11*s.* 3*d.*?

(14) £5327. 3*s.* 5*d.* ÷ £6. 13*s.* 2*d.*

(15) A man's income is £2. 7*s.* 6*d.* a-week, and his expenditure, on an average, £3. 1*s.* 10*d.*, but he has £50 to begin with. How much a-week does he spend more than he gets? How many weeks will the £50 keep him out of debt? And how much will he be in debt after 100 weeks from the commencement?

(16) A certain book cost 7¾*d.* per copy for the paper, 4½*d.* for the printing, 5½*d.* for the binding. The total issue cost £110. 18*s.* 9*d.*

Of how many copies did it consist? And what was the profit on the whole issue if each copy was sold for 2*s.*?

(17) An omnibus costs to work, 5*s.* 6*d.* a-day for the driver, 5*s.* a-day for the conductor, 8*s.* 6*d.* a-week for the keep of each of 8 horses, 6*s.* 9*d.* a-week for sundries. These omnibuses run on Sundays. If the weekly expenses amount to £333. 11*s.* 3*d.*, how many omnibuses are there at work?

(18) How many times can we subtract £1. 3*s.* 7½*d.* from £78,492, and what will be over?

(19) If I have £18. 11*s.* 9*d.*, and buy as many books as I can at 5*s.* 3*d.* each, and with the remainder buy a slate, what did it cost?

(20) With an inheritance of £12,700, I bought as many shares at £92. 7*s.* 6*d.* as I could get. How many shares at £1. 9*s.* 9*d.* can I buy with the remainder?

(21) How many times must £3. 5*s.* 8½*d.* be added to £562. 12*s.* 11*d.* to make £14,000?

(22) Find my income if my income-tax at 7*d.* in £1 amounts to £14. 2*s.* 11*d.*?

(23) How many Napoleons at 14*s.* 11½*d.* each are equal to 718 Prussian thalers at 2*s.* 10¾*d.* each?

(24) How many pounds of tea at 2*s.* 8*d.* per lb. must be given in exchange for 112 lbs. of coffee at 1*s.* 2*d.* per lb., and 88 lbs. of raw sugar at 4*d.* per lb.?

(25) £1728. 18*s.* ÷ £4. 4*s.* 9*d.*
(26) £588. 7*s.* ÷ £4. 15*s.* 8*d.*
(27) £181. 8*s.* 7½*d.* ÷ 3*s.* 7½*d.*
(28) £330 ÷ 7*s.* 4*d.*
(29) £4559. 19*s.* 6*d.* ÷ £6. 4*s.* 3*d.*
(30) £4080 ÷ £5. 13*s.* 4*d.*
(31) £93. 15*s.* ÷ 2¼*d.*
(32) £16. 10*s.* 9*d.* ÷ 2*s.* 7½*d.*
(33) £5611. 1*s.* ÷ £5. 12*s.* 4*d.*
(34) £17914. 11*s.* 11¾*d.* ÷ £7. 8*s.* 10¼*d.*
(35) £734. 8*s.* 2*d.* ÷ £7. 1*s.* 1¼*d.*
(36) £1680 ÷ £5. 7*s.* 8*d.*
(37) £154. 10*s.* ÷ 4*s.* 3¾*d.*
(38) £3157. 15*s.* 1*d.* ÷ £7. 17*s.* 7¼*d.*
(39) £601000 ÷ £823. 1*s.* 4*d.*
(40) £237. 15*s.* 6*d.* ÷ 2¾*d.*
(41) £7425. 18*s.* 10½*d.* ÷ £24. 3*s.* 8½*d.*
(42) £90197. 14*s.* 10½*d.* ÷ £81. 3*s.* 7¾*d.*
(43) £8378. 3*s.* 9½*d.* ÷ £16. 3*s.* 5¾*d.*
(44) £999900 ÷ £99. 19*s.* 11¾*d.*

CHAPTER VIII.

DIVISION (*continued*).

1. Distribute £9. 12*s.* 6*d.* equally among 3 persons. How much will each have?

The £9 will give £3 each, the 12*s.* will give 4*s.* each, and the 6*d.* will give 2*d.* each; therefore each person will have £3. 4*s.* 2*d.*

This question is indicated thus: £9. 12*s.* 6*d.* $\div$ 3.

The symbol ($\div$) has thus a perfectly *new* interpretation, viz., the distribution of a given quantity into so many *equal parts.*

Learn by heart: *This sign* ($\div$) *is called* DIVIDED BY, *and bears* TWO *interpretations; 1st.* HOW MANY TIMES *is the* QUANTITY *following the sign contained in the quantity preceding the sign, and the answer will be* SO MANY TIMES; *2nd. Distribute the quantity before the sign into as many equal parts as is indicated by the* NUMBER *after the sign, and the answer will be* SO MUCH TO EACH PART.

2. Divide £357. 3*s.* 8*d.* between 2 persons. Dividing first the 3 hundred-pound notes, each person would receive 1 hundred-pound note, and there would be 1 hundred-pound note over; this we convert into 10 ten-pound notes, which, with the 5 ten-pound notes we have already, make 15 ten-pound notes, and these, divided among the 2 persons, give 7 ten-pound notes to each, and 1 ten-pound note over; converting this into 10 pounds, and adding the 7 pounds we have already, we have 17 pounds, of which we can give 8 pounds to each of the 2 persons, leaving 1 pound over; convert this into 2 half-sovereigns, and give 1 half-sovereign to each; now divide the 3 shillings, giving 1 shilling to each, and convert the 1 shilling over into 12 pence, which, with the 8 pence, give 20 pence, of which each person will have 10 pence. Total to each person, £178. 11*s.* 10*d.*

	£.	*s.*	*d.*
2)	357	3	8
	£178	11	10

Wording: 2 in 3, **1′**, carry 1; in 15, **7′**, carry 1; in 17, **8′**, carry 1 (pound) = 2 half-sovs.; in 2, **1′**; in 3, **1′**, carry 1 (shilling) = 12 pence; in 20, **10′**. *Ans.* £178. 11*s.* 10*d.*

Divide £743. 17*s.* 8¼*d.* among 3 persons.

	£.	*s.*	*d.*
3)	743	17	8¼
	£247	19	2¾

Wording: 3 in 7, **2′**, carry 1; in 14, **4′**, carry 2; in 23, **7′**, carry 2; in 5, **1′**, carry 2; in 27, **9′**; in 8, **2′**, carry 2; in 9, **3′**. *Ans.* £247. 19*s.* 2¾*d.*

Divide £27,915. 7*s.* 8*d.* into 4 equal parts.

£.	*s.*	*d.*
4) 27915	7	8
£6978	16	11

Wording: 4 in 27, 6′, carry 3; in 39, 9′, carry 3; in 31, 7′, carry 3; in 35, 8′, carry 3; in 6, 1′, carry 2; in 27, 6′, carry 3; in 44, 11′. *Ans.* £6978. 16*s.* 11*d.*

Distribute £19007. 6*s.* 7*d.* among 7 persons.

£.	*s.*	*d.*
7) 19007	6	7
£2715	6	$7\frac{3}{4}$, $\frac{3}{4}$ over.

Wording: 7 in 19, 2′, carry 5; in 50, 7′, carry 1; in 10, 1′, carry 3; in 37, 5′, carry 2; in 46, 6′, carry 4; in 55, 7′, carry 6; in 24, 3′, and 3 farthings *over.* As we have no coin smaller than farthings, these 3 farthings must remain undistributed.

EXERCISE XXI.

(1) Distribute £17,419. 8*s.* 9*d.* between 2 persons.

(2) What is the half of £73. 13*s.* 9*d.*?

(3) Divide £68,422. 5*s.* $9\frac{3}{4}$*d.* into 3 equal parts.

(4) What is the third part of £11,019. 5*s.* 6*d.*?

(5) What sum of money can be subtracted 4 times exactly from £3509. 7*s.* 6*d.*?

(6) What is the quarter of £1007. 1*s.* 2*d.*?

(7) If 5 shares cost £7804. 6*s.* 8*d.*, what is the cost of each?

(8) What is the fifth part of £19,001. 2*s.* $7\frac{1}{4}$*d.*?

(9) If I spend £1111 in 6 years, how much is that for 1 year?

(10) What is the sixth part of £14. 9*s.* 3*d.*?

(11) What do I spend each day if I spend £19. 3*s.* 3*d.* in a week of 7 days?

(12) What is the seventh part of £85. 14*s.* $8\frac{1}{2}$*d.*?

(13) If the wages of 8 men amount to £5. 9*s.* 8*d.*, what will each receive?

(14) What is the eighth part of £19. 7*s.* 6*d.*?

(15) If 9 articles cost £8. 7*s.* $5\frac{1}{4}$*d.*, what does each cost?

(16) What is the ninth part of £318. 14*s.* $0\frac{3}{4}$*d.*?

(17) Distribute £519. 6*s.* 7*d.* equally among 3 persons.

(18) Divide £22,865. 9*s.* 11*d.* among 7 persons.

(19) What is the largest sum of money that can be subtracted 9 times from £1000?

(20) What number subtracted 8 times from £417. 6*s.* $5\frac{1}{4}$*d.* will leave a remainder of £19. 11*s.* $7\frac{1}{2}$*d.*?

(21) £43750. 8*s.* 4*d.* ÷ 2
(22) £1316. 9*s.* 7*d.* ÷ 3
(23) £5000 ÷ 4
(24) £1706. 8*s.* 6*d.* ÷ 5
(25) £1050. 10*s.* 6*d.* ÷ 6
(26) £1050. 10*s.* 6*d.* ÷ 7
(27) £1050. 10*s.* 6*d.* ÷ 8
(28) £1050. 10*s.* 6*d.* ÷ 9
(29) £473. 7*s.* 5*d.* ÷ 7
(30) £751. 1*s.* 1*d.* ÷ 8

3. Distribute 718 eggs into 2 baskets.

C. X. I.
2) 7 1 8
3 5 9 eggs.

Distributing the 7 hundreds, we put 3 hundreds into each basket, and have 1 hundred over; 1 hundred is 10 tens, and 1 ten to it makes 11 tens, which yields 5 tens for each basket, and 1 ten over, which, with the 8 units, makes 18 units, of which 9 go to each basket. *Ans.* 359 eggs to each basket.

Wording: 2 in 7, **3′**, carry 1; in 11, **5′**, carry 1; in 18, **9′**.

Arrange 329 marbles into 7 equal heaps.

7) 329
47

Ans. 47 marbles in each heap.

Arrange 329 marbles into 8 equal heaps.

8) 329
41 and 1 over.

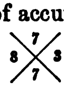
Ans. 41 marbles in each heap and 1 marble over.

Arrange 329 marbles into 9 equal heaps.

9) 329
36 and 5 over.

Ans. 36 marbles in each heap and 5 marbles over.

4. Test of accuracy by casting out nines.



8) 435265
54408 and 1 over.

Cast out nines from the dividend and place the result (7) in the upper space of the cross in the margin. Cast out nines from the divisor and the quotient, placing the results in the right and left hand spaces. Multiply these results ($8 \times 3 = 24$), and add in the

remainder (1); (24 + 1 = 25, 7), and write this in the lower space. If the figures in the upper and lower spaces do not agree, there must be an error in the work.

EXERCISE XXII.

(1) Divide 17412 things into 2 equal parts.

(2) What is the half of 358 things?

(3) Divide 71235 things into 3 equal parts.

(4) What is the third part of 72861?

(5) What number of things can be subtracted four times exactly from 711136 things?

(6) What is the quarter of 1097324?

(7) If 5 equal baskets contain together 3125 apples, what will 1 basket contain?

(8) What is the fifth part of 3116845?

(9) Divide a journey of 528 miles into 6 equal stages.

(10) What is the sixth part of 2034?

(11) A certain railway guard travels 2303 miles a-week. How much is that a-day?

(12) What is the seventh part of 5565?

(13) If 8 horses can draw a load of 3416 lbs., what can 1 horse draw?

(14) What is the eighth part of 137904?

(15) If 9 regiments contain 8433 men, how many are there in each?

(16) What is the ninth part of 1000008?

(17) Distribute 92763 cartridges among 11 regiments.

(18) What is the eleventh part of 135795?

(19) If 12 volumes have 6084 pages, how many are there in each?

(20) Find the twelfth part of 122436.

(21) Divide 7034519 separately by 2, 3, 4, 5, 6, 7, 8, 9, 11, 12.

(22) Divide each of the following numbers by 12:

a. 1000000	*d.* 1111111	*g.* 106000
b. 4197641	*e.* 167625	*h.* 17000000
c. 42050000	*f.* 4065000	*k.* 9052500

(23) What sum of money must be multiplied by 2 to yield £317. 19s. 6d.?

(24) What number must be multiplied by 2 to give 173598?

(25) What sum of money must be taken 3 times to yield £1000?

(26) What number must be repeated 3 times to give 14367?

(27) What sum of money multiplied by 4 will amount to £1719. 3s. 6d.?

(28) What number multiplied by 4 will give 17003068?

(29) What was the value of each collection, if 5 collections yielded £317. 16s. $10\frac{1}{2}$d.?

(30) How long is each side of a regular pentagon, if the whole perimeter is 935 inches?

(31) What sum of money must be multiplied by 6 to yield £4379?

(32) What number must be taken 6 times to give 14382?

(33) What is that sum of money which multiplied by 7 gives £962. 5s. $0\frac{1}{2}$d.?

(34) Find the number which multiplied by 7 yields 100569.

(35) What sum of money repeated 8 times will yield £19,000?

(36) What number multiplied by 8 will give 5371016?

(37) What sum of money multiplied by 9 will amount to £384. 15s. $11\frac{1}{4}$d.?

(38) What number multiplied by 9 will give 123456789?

(39) What sum of money multiplied by 11 will give £38,020. 4s. $9\frac{1}{2}$d.?

(40) What number taken 12 times will give 207000?

5. *Q.* What is the eighth part of 24 apples?

A. 3 apples.

Q. Why?

A. Because there are 8 threes in 24.

Q. How many times are 8 apples contained in 24 apples?

A. 3 times.

Q. Why?

A. Because there are 3 eights in 24.

Teacher. If, therefore, 24 is divided by 8, the quotient will be 3, whichever of the two interpretations (§ 1) of the sign ($\div$) we adopt; but the name to be attached to the quotient will depend on the interpretation required by the nature of the question; thus here the answer is in one case 3 *apples*, and in the other 3 *times*. In future, then, we may say, $24 \div 8 = 3$, without necessarily choosing between the two interpretations.

6. Distribute £718,436. 3*s.* 9¼*d.* amongst 317 persons.

If we apply the reasoning of § 2, we meet at once with the practical difficulty that we only know the multiplication table up to twelves; we therefore make a multiplication table for this occasion, thus:

$$
\begin{array}{r}
1 \times 317 = 317 \\
2 \times 317 = 634 \\
3 \times 317 = 951 \\
4 \times 317 = 1268 \\
5 \times 317 = 1585 \\
6 \times 317 = 1902 \\
7 \times 317 = 2219 \\
8 \times 317 = 2536 \\
9 \times 317 = \underline{2853} \\
14265
\end{array}
$$

N.B. The accuracy of the table may be tested by adding the nine products, and also multiplying the last product (the 9 times) by 5; these two results should agree.

Successive stages of the sum.

```
     CM.XM.M.C.X.I.  S.  D.F.
317) 7  1 8 4 3 6   3   9 ¼ (

                           M.
317) 718436   3   9¼ (2
     634
      84

                          M.C.
317) 718436   3   9¼ (2 2
     634
      844
      634
      210
```

If 7 hundred-thousand-pound notes are to be distributed among 317 persons, there will not be a note a-piece; we therefore change them into notes of ten-thousand pounds, of which the 7 will yield 70, and 1 more which we have makes 71; these again will not yield 1 ten-thousand-pound note to each person. Converting these into thousand-pound notes, we obtain, with the 8, 718 thousand-pound notes; now we see by the table that twice 317 is less than 718, but that 3 times 317 is greater than 718, we therefore can give 2 thousand-pound

uccessive stages of the sum.

M.C.X.
17) 718436 3 9¼ (2 2 6
634
844
634
2103
1902
201

M.C.X.I.
317) 718436 3 9¼(2 2 6 6
634
844
634
2103
1902
2016
1902
114
2
228

M.C.X.I. £½.S. D.
317) 718436 3 9¼ (2 2 6 6 — 7 2¼
634
844
634
2103
1902
2016
1902
114
2
2283
2219
64
12
768
9
777
634
143
4
572
1
573
317
256

notes to each person, and shall have *some* thousand-pound notes over. Twice 317 = 634; subtracting this number from 718, leaves 84 thousand-pound notes, which, with the 4 hundred-pound notes, yield 844 hundred-pound notes, of which, by similar reasoning, we can give 2 hundred-pound notes to each, leaving us 210 hundred-pound notes over. Converting these into ten-pound notes, we obtain, with the 3, 2103 ten-pound notes. Consulting the table, we find we can give each person 6 ten-pound notes, and subtracting 6 × 317, or 1902, we have 201 ten-pound notes over. These, with the 6 pounds, make 2016 pounds, enabling us to give 6 pounds to each person, and leaving 114 pounds. To distribute these, we must change them into smaller coin, viz. half-sovereigns. Since each pound equals 2 half-sovs., multiply 114 by 2, yielding 228 half-sovs., which, being less than 317, must be again changed into shillings, and, with the 3, will yield 2283 shillings. We can now give each person 7 shillings, leaving 64 shillings. These changed into pence will yield 12 × 64 = 768 pence, and adding in the 9*d.*, 777 pence. Of these, we can give 2 pence to each person, leaving 143 pence. Multiplying these by 4 to convert them into farthings, and adding in the 1 farthing we have already, we get 573 farthings, of which we can give 1 farthing to each person, leaving 256 farthings. This remainder must be left undistributed. Hence the answer is, £2266. 7*s.* 2¼*d.* to each person, and 256 farthings over.

Table.

$1 \times 137 =$	137
$2 \times 137 =$	274
$3 \times 137 =$	411
$4 \times 137 =$	548
$5 \times 137 =$	685
$6 \times 137 =$	822
$7 \times 137 =$	959
$8 \times 137 =$	1096
$9 \times 137 =$	1233
	6165

Second meaning.

7. Distribute 48729 marbles among 137 people. How many *marbles* to each? 1st. Make a table as before. 2nd. Analyse the dividend: 48729 = 4 ten thousands and 8729 units over = 48 thousands and 729 units over = 487 hundreds and 29 units over = 4872 tens and 9 units over (p. 7). 3rd. Proceed as follows: If 4 ten thousands are to be distributed amongst 137 persons, there will not be 1 ten thousand a-piece; the 48 thousands also will not yield 1 thousand a-piece. If the 487 hundreds are distributed amongst the 137 persons, we can give (see table) 3 hundreds a-piece, but not 4; subtracting 3 times 137 = 411 (see table) from 487 leaves 76 hundreds = 760 tens, which, with the 2 tens we have already, make 762 tens; of these, we can give (see table) 5 tens to each, leaving 77 tens = 770 units, which with the 9 units we have already, make 779 units; of these we can give (see table) 5 units to each,

Successive stages of the sum.

```
        C.
137) 48729 (3
     411
      76
```

```
        C.X.
137) 48729 (35
     411
      762
      685
       77
```

First meaning.

How many *times* are 137 marbles contained in 48729 marbles? 1st. Make the same table as in the margin. 2nd. Proceed as follows: We wish to take out at one subtraction as many times 137 as we can. It is clear that we can take 10×137 or 1370; we can also take it 100 times or 13700, but not 1000 times, for $1000 \times 137 = 137000$, which is more than the dividend. Now the question arises, Can we take more than 1 hundred at one subtraction? From the table, we see that $200 \times 137 = 27400$, $300 \times 137 = 41100$, but $400 \times 137 = 54800$, which is larger than the dividend. Subtracting, therefore, 300×137, we have 7629 units over.

```
137) 48729 (300 times
     41100
      7629 over
```

Out of this remainder we again wish to take at one subtraction as many times 137 as we can. We cannot take it another hundred times, or else we could have taken it 400

Successive stages of the sum.

```
                C.X.I.
137) 48729 (3 5 5
     411
     ---
      762
      685
      ---
       779
       685
       ---
        94
```

leaving 94 units over. The answer, therefore, is 355 *marbles* to each person, and 94 marbles over.

times at the last subtraction. By reasoning as above, we see from the table that we can take it 50 times.

```
137) 48729
     41100 = 300 times
     -----
      7629
      6850 -  50 times
      ----
       779 over
```

From the table, we see again that we can take it out 5 times more.

```
137) 48729
     41100 = 300 times
     -----
      7629
      6850 =  50 times
      ----
       779
       685 =   5 times
       ---     ---
Remainder, 94  355 times
```

Casting out nines:

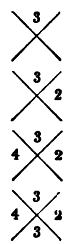
4, 12, 3, 10, 1, 3′ ....

1, 4, 11, 2′ ....

3, 8, 13, 4.........................

Remainder, 4′ ....

2 × 4 = 8 ; 8, 12, 3′ ....



Ans. 355 *times* and 94 marbles over.

On comparing these two processes, we notice (1st) that the numerical values of the quotient and remainder are the same in each, the only difference being in the *meaning* of the quotient, as we might have expected from § 5; (2nd) that the form under the second meaning uses fewer figures. We shall adopt this method in all cases of division, except where the divisor is a compound number, in which case we must follow the method of Chapter VII. We must, however, take especial care to interpret the quotient rightly, and in this we can only be guided by the answer demanded in the question.

8. Divide 1876706 by 7598.

Table.

```
1 × 7598 =  7598
2 × 7598 = 15196
3 × 7598 = 22794
4 × 7598 = 30392
5 × 7598 = 37990
6 × 7598 = 45588
7 × 7598 = 53186
8 × 7598 = 60784
9 × 7598 = 68382
           ------
           341910
```

```
                    C.X.I.
7598) 1876706 (2 4 7
      15196
      -----
       35710
       30392
       -----
        53186
        53186
        -----
        — —
```

```
7598) 1876706
      1519600 = 200 times
      -------
       357106
       303920 =  40 times
       ------
        53186
        53186 =   7 times
        -----   ---
        — —     247 times exactly
```

Therefore, if 1876706 things are distributed into 7598 equal lots, there will be 247 things in each lot. Again, if 1876706 things be distributed into lots of 7598 things each, there will be 247 lots, or, which is the same thing, $7598 \times 247 = 1876706$.

$$
\begin{array}{r}
7598 \\
247 \\
\hline
1519600 \\
303920 \\
53186 \\
\hline
1876706
\end{array}
$$

Verification:

Comparing this multiplication with the division above, we see that the several addenda of the one are the successive subtrahends of the other, as might have been anticipated, seeing that Multiplication is shortened Addition, and Division is shortened Subtraction.

In such verification the remainder, if any, must be added to the product. Thus, to verify the first sum in this section :

$$
\begin{array}{rr}
 & 137 \\
 & 355 \\
\hline
 & 41100 \\
 & 6850 \\
 & 685 \\
\hline
 & 48635 \\
\text{Remainder,} & 94 \\
\hline
 & 48729
\end{array}
$$

Divide 14634167 by 3578.

$$
\begin{array}{l}
1 \times 3578 = 3578 \\
2 \times 3578 = 7156 \\
3 \times 3578 = 10734 \\
4 \times 3578 = 14312 \\
5 \times 3578 = 17890 \\
6 \times 3578 = 21468 \\
7 \times 3578 = 25046 \\
8 \times 3578 = 28624 \\
9 \times 3578 = 32202 \\
\hline
\qquad\quad 161010
\end{array}
\qquad
\begin{array}{l}
\qquad\qquad\qquad\quad \text{M.C.X.I.} \\
3578)\ 14634167\ (4\,0\,9\,0 \\
\qquad\ \ 14312 \\
\qquad\ \ \overline{\quad 32216} \\
\qquad\quad\ \ 32202 \\
\qquad\quad\ \ \overline{\qquad 147}
\end{array}
$$

Ans. 4090 times or things, and 147 things over.

N.B. After the first subtraction, we find in "bringing down" the 1 hundred that 3578 is not contained in 3221 ; we must therefore be very careful to register this in the quotient by putting a 0 in the hundreds' place. Similarly, after the

second subtraction, we find that there are no units, and we again put a 0 in the units' place of the quotient. This remark shews the importance of not discarding too soon the headings, I. X. C., &c. When considerable facility has been acquired, these, as well as the marginal table, may be omitted. In the method given under the first meaning (p. 90), this difficulty would not have arisen.

Exercise XXIII.

(1) Distribute £17,802. 19*s.* 6¼*d.* among 371 persons..

(2) What sum of money must be multiplied by 135 to yield £2801. 10*s.* 7½*d.*?

(3) Distribute £643. 10*s.* 2½*d.* among 815 persons.

(4) What sum of money can be subtracted 750 times exactly from £706. 5*s.*?

(5) £6043. 11*s.* 7*d.* ÷ 892.

(6) If I spend £507. 19*s.* 2*d.* a-year, how much is that a-day?

(7) If I spend £1068. 5*s.* 6*d.* a-year, how much is that a-week?

(8) Divide £7003. 3*s.* 10¾*d.* into 1867 equal parts.

(9) If I invest £53,833. 5*s.* in 1234 shares, what is that per share?

(10) If I pay £323. 18*s.* 1*d.* for 764 pieces of calico of 37 yards per piece, how much does each piece cost, and how much a yard?

(11) 83094027 ÷ 9784.

(12) 140940000 ÷ 13417.

(13) 140940000 ÷ 130417.

(14) 8375808125 ÷ 18368.

(15) 22505600 ÷ 17312.

(16) 3491938567017 ÷ 857928.

(17) How many bags of 819 marbles can I fill out of 20000 marbles?

(18) Divide the sum of 71 × 2117 and 17 × 1711 by 29.

(19) Divide the difference between the same quantities by 58.

(20) What sum of money subtracted 94 times from £848. 6*s.* 5*d.* will leave £62. 4*s.* 11*d.*?

(21) How much a-week may I spend out of an income of £374. 12*s.* 2*d.* a-year, to save 75 guineas a-year?

(22) £118,043. 19*s.* 7¼*d.* ÷ 269.

(23) £1000000 ÷ 89.
(24) £1000000 ÷ 267.
(25) £1000000 ÷ 1869.
(26) £1000000 ÷ 9345.
(27) £1000000 ÷ 84105.
(28) £1000000 ÷ 925155.

(29) If 4204800 ounces of provisions are supplied to a regiment of 960 men, giving 12 ounces a-day to each man, how long will the supply last?

(30) How long would it last if each man had 60 ounces a-day?

(31) If the regiment were reduced to 800 men, each man having 24 ounces a-day, how long would it last?

(32) If at the age of 29 I begin business with a capital of £4500, and wish to retire at the age of 60 with a capital of £20,000, what yearly addition must I make to my capital?

(33) If a million bricks be required, and we have 77331 already, how many loads of 407 bricks each are wanted to complete the number?

(34) What will be the charge for translating 25344 words, at the rate of 1*s.* 7½*d.* per folio of 72 words?

(35) 248073019 ÷ 43017.

(36) 943867315 ÷ 12604.

(37) I sold 1512 articles for £690. 7*s.* 6*d.*, making a total profit of £123. 7*s.* 6*d.* Find the cost of each.

(38)* What sum of money must be multiplied by 623 to yield £733. 6*s.* 5½*d.*?

(39)* What sum of money must be multiplied by 1246 to yield £733. 6*s.* 5½*d*?

(40)* What sum of money must be multiplied by 623 to yield £366. 13*s.* 2¾*d*?

(41) 12357096 ÷ 419
(42) 214583206 ÷ 42576
(43) 366413796 ÷ 45796
(44) 166944509 ÷ 23509
(45) 19639 ÷ 23
(46) 39278 ÷ 46
(47) 20969 ÷ 13
(48) 100004 ÷ 17
(49) 1010100 ÷ 19
(50) 1000000 ÷ 37
(51) 3000000 ÷ 111

* Examine and compare the quotients in these three.

Exercise XXIV.

(1) Find the sum of 578, 364, 927, 9768.

(2) What number exceeds 578 by 344?

(3) From what sum of money must £42. 11*s*. 8*d*. be deducted to leave £57. 8*s*. 4*d*.?

(4) There are two numbers; the less is 7109, and their difference is 591. Find the greater.

(5) From what number must 316 be taken away to leave 518?

(6) If from a certain number 75 is taken, 89 is left. Find the number.

(7) Of two partners A and B, A contributes £520 less than B, whose share is £965. Find the total capital.

(8) 493 exceeds a certain number by 121. Find the number.

(9) What number falls short of 1096 by 421?

(10) What number is that to which 2768 must be added to give 10000.

(11) There are two numbers; the greater is 16520, and their difference is 3736. Find the less.

(12) What number increased by 2743 becomes 12000?

(13) Find the product of 69 and 237.

(14) What number contains 328 exactly 328 times?

(15) From what number can 5704 be taken exactly 104 times?

(16) From what number can 847 be taken 307 times, leaving a remainder of 49?

(17) Of what number is 53 the 7th part?

(18) What number divided by 97 gives 204?

(19) What is the 235th part of 141235?

(20) By what number must 397 be multiplied to give 170710?

(21) The product of two numbers is 4539, one factor is 51. Find the other.

(22) Given divisor 9373, quotient 103. Find the dividend.

(23) Given dividend 9373, quotient 103. Find divisor.

(24) Given dividend 9373, divisor 103. Find quotient.

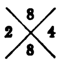
(25) What number taken 103 times gives 965419?

(26) Given dividend 99201815, quotient 208, remainder 1207. Find divisor.

CHAPTER IX.

CONTRACTED OPERATIONS.

1. Multiply 5879 by 364.

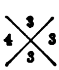


```
      Full form.              Contracted form.
         5879                      5879
          364                       364
      -------                  --------
      1763700                  17637
       352740                   35274          a
        23516                    23516
      -------                  --------
      2139956                  2139956

or,      5879         or,          5879
          364                       364
      -------                  --------
        23516                    23516
       352740                   35274          b
      1763700                  17637
      -------                  --------
      2139956                  2139956
```

By carefully comparing the full with the contracted forms, it will be seen that the effect of the ciphers to the right of the several products can be produced by merely leaving blanks in their places. The form most commonly used is that marked *b*, but that marked *a* may also be practised with advantage.

Multiply 480730 by 5070.



```
      Full form.              Contracted form.
           480730                  480730
             5070                    5070
      -----------              ----------
      2403650000               2403650
        33651100                 33651100
      -----------              ----------
      2437301100               2437301100

or,        480730     or,          480730
             5070                    5070
      -----------              ----------
        33651100                 33651100
      2403650000               2403650
      -----------              ----------
      2437301100               2437301100
```

Should any difficulty be experienced, the places of the ciphers may at first be indicated by dots, thus:

387490 × 20400.


```
  387490
   20400
---------
154996000
774980....
----------
7904796000
```

It is usual not to omit the ciphers of the multiplicand, nor those arising from ciphers at the end of the multiplier. But this process should be still further contracted, thus:

```
  38749(0
    204(00
----------
 154996
77498
----------
7904796000
```

In this case we have multiplied only the significant figures, affixing to the ultimate product the final ciphers of both the factors, the reason for which follows directly from Ch. VI. § 5.

Exercise XXV.

(1) 8417 × 394
(2) 27349 × 5618
(3) 108912 × 4798
(4) 247863 × 365
(5) 68354 × 842
(6) 92731 × 516
(7) 430597 × 118
(8) 2413058 × 4237
(9) 48596 × 3420
(10) 68043 × 5070
(11) 4235700 × 8005
(12) 73050 × 9010
(13) 20014 × 1050
(14) 68000000 × 45000
(15) 2076980 × 30840
(16) 385604000 × 10500
(17) 30420 × 103684700
(18) 437598000 × 4601700
(19) 2804 × 43090
(20) 5498600 × 6420
(21) 6200 × 70800 × 9500
(22) 5860 × 2045 × 902 × 1000

2. Simplify 4723 × 8 + 1257.

Full form.

```
 4723
    8
-----
37784
 1257
-----
39041
```

Contracted form.

$\dot{4}\dot{7}\dot{2}\dot{3} + \dot{1}\dot{2}\dot{5}\dot{7}$

```
    8
-----
39041
```

Wording of contracted process at full length: 8 times $\dot{3}$ are 24, and $\dot{7}$ are 31, put down 1 and carry 3; 8 times $\dot{2}$ are 16, and 3 are 19, and $\dot{5}$ are 24, put down 4 and carry 2; 8 times $\dot{7}$ are 56, and 2 are 58, and $\dot{2}$ are 60, put down 0 and carry 6; 8 times $\dot{4}$ are 32, and 6 are 38, and $\dot{1}$ are 39, put down 39.

Wording to be used: 24, 31′, carry 3; 16, 19, 24′, carry 2; 56, 58, 60′, carry 6; 32, 38, 39′.

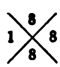
It will be found a safeguard against error to dot each figure as it is used.

EXERCISE XXVI.

(1) Simplify 57839 × 7 + 17094.
(2) Simplify 1703048 × 6 + 72496.
(3) Simplify 437068109 × 5 + 942578631.
(4) Simplify 91827364 × 8 + 42596824.
(5) Simplify 803768915 × 9 + 16590612.
(6) Simplify 974823516 × 4 + 987654321.

3. Multiply 68437 by 17.

Full form.	Contracted form.
68437	$\dot{6}\dot{8}\dot{4}\dot{3}\dot{7}$. × 17
17	1163429
68437.	
479059	
1163429	



Wording in full: 7 times $\dot{7}$ are 49, put down 9 and carry 4; 7 times $\dot{3}$ are 21, and 4 are 25, and $\dot{7}$ are 32, put down 2 and carry 3, &c.

Wording to be used: 49′, carry 4; 21, 25, 32′, carry 3; 28, 31, 34′, carry 3; 56, 59, 63′, carry 6; 42, 48, 56′, carry 5; 11′.

EXERCISE XXVII.

(1) 438592 × 17
(2) 6017839 × 16
(3) 60857410 × 15
(4) 79845103 × 18
(5) 142587623 × 13
(6) 98043527 × 14
(7) 3260625 × 16
(8) 54370650 × 19
(9) 58472 × 13
(10) 8345620 × 17
(11) 287546 × 15
(12) 7863521 × 18
(13) 23781 × 14
(14) 4236541 × 17
(15) 1835429 × 13
(16) 987654 × 16
(17) 320070 × 1400
(18) 100936 × 17000
(19) 10608000 × 1300
(20) 234500 × 16000

4. Multiply 358934 by 103.

Full form.

358934
103
―――――
358934..
1076802
―――――
36970202

Contracted form.

$\dot{3}\dot{5}\dot{8}\dot{9}\dot{3}\dot{4}$.. × 103
―――――
36970202

Wording: **12′**, carry 1; 9, **10′**, carry 1; 27, 28, **32′**, carry 3; 24, 27, **30′**, carry 3; 15, 18, **27′**, carry 2; 9, 11, **19′**, carry 1; **6′**; **3′**.

Multiply 4206008 by 10004.

Full form.

4206008
10004
―――――
4206008....
16824032
―――――
42076904032

Contracted form.

$\dot{4}\dot{2}\dot{0}\dot{6}\dot{0}\dot{0}\dot{8}$.... × 10004
―――――
42076904032

Wording: **32′**, carry 3; 0, **3′**; **0′**; **24′**, carry 2; 0, 2, **10′**, carry 1; 8, **9′**; **16′**, carry 1; **7′**; **0′**; **2′**; **4′**.

Multiply 6749840 by 10600.

Full form.

674984(0
106(00
―――――
674984..
4049904
―――――
71548304000

Contracted form.

$\dot{6}\dot{7}\dot{4}\dot{9}\dot{8}\dot{4}$0.. × 10600
―――――
71548304000

Wording: **24′**, carry 2; 48, **50′**, carry 5; 54, 59, **63′**, carry 6; 24, 30, **38′**, carry 3; 42, 45, **54′**, carry 5; 36, 41, **45′**, carry 4; **11′**, carry 1; **7′**. Now put on the three ciphers.

Exercise XXVIII.

(1) 7563124 × 101	(11) 58432631 × 108
(2) 56342901 × 1001	(12) 857320507 × 1005
(3) 290076358 × 10001	(13) 25603054 × 1008
(4) 624345 × 102	(14) 8946237 × 19
(5) 82654352 × 103	(15) 8946237 × 109
(6) 9432654 × 107	(16) 8946237 × 1009
(7) 20560078 × 1007	(17) 27438956 × 106
(8) 254836241 × 104	(18) 537609000 × 1020
(9) 58364212 × 111	(19) 29330 × 170 × 1070 × 10070
(10) 246314 × 1011	(20) 358740100 × 10800

5. Multiply 437062 by 21.

Full form.	Contracted form.
437062	$\dot{4}\dot{3}\dot{7}\dot{0}\dot{6}\dot{2} \times 21$
21	9178302
437062	
874124.	
9178302	

Wording in full: Once 2 is 2′; twice $\dot{2}$ are 4, and 6 are 10′, carry 1; twice $\dot{6}$ are 12, and 1 are 13, and $\dot{0}$ are 13′, carry 1; twice $\dot{0}$ is 0, and 1 is 1, and $\dot{7}$ is 8′; twice $\dot{7}$ are 14, and $\dot{3}$ are 17′, carry 1; twice $\dot{3}$ are 6, and 1 are 7, and $\dot{4}$ are 11′, carry 1; twice $\dot{4}$ are 8, and 1 are 9′.

Wording to be used: 2′; 4, 10′, carry 1; 12, 13′, carry 1; 0, 1, 8′; 14, 17′, carry 1; 6, 7, 11′, carry 1; 8, 9′.

Exercise XXIX.

(1) 52019763 × 21
(2) 68320094 × 31
(3) 4107618 × 41
(4) 5090090 × 51
(5) 16487312 × 61
(6) 43506 × 71
(7) 728943 × 81
(8) 19312650 × 91
(9) 32863400 × 610
(10) 52900 × 4100
(11) 12468000 × 71000
(12) 48760200 × 3100
(13) 27341 × 71
(14) 27341 × 17
(15) 376859 × 31
(16) 376859 × 13
(17) 419068 × 15
(18) 419068 × 51
(19) 206867 × 41
(20) 206867 × 14

6. Multiply 7849624 by 6001.

Full form.	Contracted form.
7849624	$\dot{7}\dot{8}\dot{4}\dot{9}\dot{6}\dot{2}\dot{4} \times 6001$
6001	47105593624
7849624	
47097744...	
47105593624	

Wording in full: 4′; 2′; 6′; 6 times $\dot{4}$ are 24, and $\dot{9}$ are 33′, carry 3; 6 times $\dot{2}$ are 12, and 3 are 15, and $\dot{4}$ are 19′, carry 1, &c.

Wording to be used: 4′; 2′; 6′; 24, 33′, carry 3; 12, 15, 19′, carry 1; 36, 37, 45′, carry 4; 54, 58, 65′, carry 6; 24, 30′, carry 3; 48, 51′, carry 5; 42, 47′.

Exercise XXX.

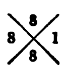
(1) 35943628 × 201
(2) 9016706 × 301
(3) 11043690 × 401
(4) 763005090 × 501
(5) 2109863 × 6001
(6) 2109863 × 601
(7) 2109863 × 61
(8) 21098630 × 60001
(9) 475328956 × 7010
(10) 325408600 × 8010
(11) 325408600 × 810
(12) 325408600 × 180
(13) 104578000 × 901
(14) 104578000 × 1090
(15) 478006043 × 17
(16) 478006043 × 71
(17) 478006043 × 701
(18) 478006043 × 107
(19) 478006043 × 1007
(20) 478006043 × 7001

7. Multiply 48967 by 742.

```
  48967
    742
 ------
342769.. A
 2056614 B
 --------
36333514
```

Line A is 7 times the multiplicand; line B is 6 times line A, and being placed two figures to the right of line A, gives to the line A its required value, viz. 700 times the multiplicand.

Multiply 47213 by 568.



```
  47213
    568
 ------
 377704  A
2643928  B
--------
26816984
```

Line A is 8 times the multiplicand; line B is 7 times line A, or 56 times the multiplicand, and being written one place to the left, has the required value, viz. 560 times.

Similarly we can multiply by 1768 or 6817 in two lines, by noticing that 68 = 4 × 17. Multiply by 17 in one line, as in § 3.

This process may be still further contracted by adding the figures of the second product as fast as they are obtained to those of the first product.

```
 47213 × 568
 -----------
  377704
 --------
26816984
```

Wording of last line: **4′**; **28′**, carry 2; 0, 2, **9′**, 49, **56′**, carry 5; 49, 54, **61′**, carry 6; 49, 55, **58′**, carry 5; 21, **26′**.

Exercise XXXI.

(1) 48927653 × 742
(2) 48927653 × 427
(3) 1093856 × 545
(4) 3256740 × 5450
(5) 18094300 × 8240
(6) 4760095 × 248
(7) 37823500 × 76300
(8) 42050000 × 6370
(9) 517328645 × 1872
(10) 4072908 × 5614
(11) 15827632 × 780130
(12) 243598000 × 1909500
(13) 357823492 × 50350
(14) 4768923150 × 720900
(15) 329616000 × 17068
(16) 52631578967 × 680017
(17) 89777630050 × 816
(18) 57234000905 × 954
(19) 853011009375 × 107214
(20) 46371968750 × 16480

8. We must now once more revert to Subtraction, and adopt another mode of reasoning and another form of words.*

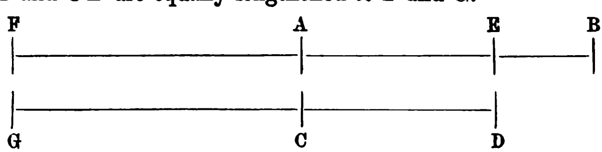
Find the difference between 430172 and 189567.

THE DIFFERENCE BETWEEN TWO NUMBERS WILL NOT BE ALTERED IF THE SAME QUANTITY IS ADDED TO BOTH OF THEM.

Illustrations: "Conceive two baskets with pebbles in them, in the first of which are 100 pebbles more than in the second. If I put 50 more pebbles into each of them, there are still only 100 more in the first than in the second."—*De Morgan's Arithmetic.*

The difference in age between a parent and his child will remain the same through life.

The difference, E B, between the lines A B and C D is not altered if A B and C D are equally lengthened to F and G.



CM.	XM.	M.	C.	X.	I.		
	10	10	10		10		
4	3	0	1	7	2	8	Minuend
1	8	9	5	6	7	0	Subtrahend
1	1	1		1			
2	4	0	6	0	5	8′	Difference

* It was not without some regret and much reflection that we did not at once in Chapter IV. adopt the method here given. An experience of upwards of twenty years has convinced us that the rationale of this process cannot be properly realized at the early stage at which Subtraction is necessary.

7 units cannot be taken from 2 units; we may add ten units to the minuend, provided we add the same, or its equivalent, 1 ten, to the subtrahend. Do so; and we have now to take 7 units from 12 units, leaving 5 units. In the tens' column we have now to subtract, not 6, but 7 tens; 7 tens from 7 tens leaves 0 tens; 5 hundreds, again, cannot be taken from 1 hundred; add 10 hundreds to the minuend, and its equivalent, 1 thousand, to the subtrahend. 5 hundreds from 11 hundreds leave 6 hundreds. Again, 10 thousands (not 9 thousands) cannot be taken from 0 thousands; add 10 thousands to the minuend, and its equivalent, 1 ten-thousand, to the subtrahend; 10 thousands from 10 thousands leave 0 thousands. Again, 9 ten-thousands (not 8) cannot be taken from 3 ten-thousands; add 10 ten-thousands to the minuend, and its equivalent, 1 hundred-thousand to the subtrahend; 9 ten-thousands from 13 ten-thousands leave 4 ten-thousands. 2 hundred-thousands (not 1) from 4 hundred-thousands leave 2 hundred-thousands.

Find the difference between £502. 1*s.* 2*d.* and £138. 17*s.* 8¼*d.*

£.			£½.	*s.*	*d.*		
C.	X.	I.					
	10	10		20	12	4/4	
5	0	2		1	2		Minuend
1	3	8	1	7	8	¼	Subtrahend
1	1	1		1	1		
3	6	3		3	5	¾	Difference

1 farthing cannot be taken from 0; add 4 farthings to the minuend and 1 penny to the subtrahend; now 1 farthing from 4 farthings leaves 3 farthings. 9 (not 8) pence from 2 pence, we cannot; add 12 pence to the minuend and 1 shilling to the subtrahend; 9 from 14, 5 pence; 18 (not 17) from 1, we cannot; add 20 shillings to the minuend and £1 to the subtrahend; 18 from 21, 3 shillings. £9 (not 8) from £2 we cannot; add £10 to the minuend and 1 ten-pound note to the subtrahend; 9 from 12 leaves £3. Similarly 4 ten-pound notes from 10 ten-pound notes leave 6 ten-pound notes; 2 hundreds from 5 hundreds leave 3 hundreds.

9. The question, "7 from 9 leaves *what*?" is equivalent to the question, "7 and *what* makes 9?" *Answer.* 7 and 2′ = 9. For the processes we have in view this mode of expression must be adopted.

Subtract 8107214 from 9689476.

9689476	4	*Wording:* 4 *and* **2′** are 6; *and* **6′** are 7; 2 *and* **2′** are 4; 7 *and* **2′** are 9; 0 *and* ′are 8; 1 *and* **5′** are 6; 8 *and* **1′** are 9.
8107214	5	
1582262	**8′**	

N.B. The figures after "and" must be written down *in the act of utterance.*

Subtract 176896 from 3401023.

3401023	4	*Wording:* 6 *and* **7′** are 13, carry 1; 10 *and* **2′** are 12, carry 1; 9 *and* **1′** are 10, carry 1; 7 *and* **4′** are 11, carry 1; 8 *and* **2′** are 10, carry 1; 2 *and* **2′** are 4; **3′**.
176896	1	
3224127	**3′**	

EXERCISE XXXII.

Work Exercise VII., using this wording.

10. Instead of uniformly adding 10 to both minuend and subtrahend, we might add any other number we pleased.

Subtract 1879642 from 3002161.

M².	CM.	XM.	M.	C.	X.	I.	
	10	20	80	50	40	30	
3	0	0	2	1	6	1	Minuend
1	8	7	9	6	4	2	Subtrahend
1	2	8	5	4	3		
1	0	5	68	41	39	29	

This answer must be correct, as we have committed no error in the reasoning; but, as it is inconvenient, we must alter its form, converting the 29 units into tens, the tens into hundreds, and so on.

29 units = **9′** units and 2 tens, which with the 39 tens = 41 tens = **1′** ten and 4 hundreds; (41 + 4) hundreds = 45 hundreds = **5′** hundreds and 4 thousands; (68 + 4) thousands = 72 thousands = **2′** thousands and 7 ten-thousands; (7 + 5) ten-thousands = 12 ten-thousands = **2′** ten-thousands and 1 hundred-thousand; (1 + 0) hundred-thousand = 1′ hundred-thousand, and there is 1′ million. *Answer.* 1122519.

The same answer will be obtained by the usual method.

3002161
1879642
1122519

11. Subtract the sum of 683, 1279, 43080, 1241596, 386724, and 427689, from 32684215 in one operation.

32684215	4		Minuend
683	8	}	Addenda and Subtrahend.
1279	1		
43080	6		
1241596	1		
386724	3		
427689	0		
30583164	3′		

The units of the subtrahend amount to 31, which cannot be taken from the 5 of the minuend; the most convenient number to be added to both minuend and subtrahend is evidently 30 units and 3 tens respectively. 31 and 4′ are 35; we therefore put down 4 and add or carry 3 to the next column of the subtrahend, and so on.

Wording: 9, 13, 19, 28, 31, *and* 4′ are 35, carry 3; 11, 13, 22, 30, 37, 45, *and* 6′ are 51, carry 5; 11, 18, 23, 25, 31, *and* 1′ are 32, carry 3; 10, 16, 17, 20, 21, *and* 3′ are 24, carry 2; 4, 12, 16, 20, *and* 8′ are 28, carry 2; 6, 9, 11, *and* 5′ are 16, carry 1; 2 *and* 0′ are 2; 3′.

EXERCISE XXXIII.

(1) From 5019308 take
62412
127842
5708
13052
58009
417925

(2) From 3785926 take
408620
7053
12019
38727
1968
423016

(3) From 348009052 take
7016094
14802060
24768923
5600082
28004015
13017019

(4) From 18050000 take
857142
857142
857142
857142
857142

(5) From 7009013 take
68459
194360
4011318
604098
28943
173666

(6) From 53685947 take
6078254
6078254
6078254
6078254
6078254
6078254

(7) From 5000000 take
18049
7683
354975
867324
841825
1019078

(8) From 10000000 take
345978
345978
345978
345978
345978
345978
345978
345978

(9) From 62947 take
6083
7509
1234
8765
9012
10010

(10) From 17685924 take
946877
946877
946877
946877
946877
946877
946877
946877
946877

12. Subtract 7×35698 from 310049.

310049	Minuend
35698×7	Subtrahend
60163	

Wording: 56 *and* 3′, 59, carry 5; 63, 68, *and* 6′, 74, carry 7; 42, 49, *and* 1′, 50, carry 5; 35, 40, *and* 0′, 40, carry 4; 21, 25, *and* 6′, 31, carry 3; 3 *and* 0, 3.

EXERCISE XXXIV.

(1) $2530168 - 137519 \times 7$
(2) $40097123 - 240368 \times 2$
(3) $711160358 - 286709 \times 9$
(4) $4986342 - 111111 \times 8$
(5) $1000000 - 142857 \times 7$
(6) $6666329 - 700000 \times 3$
(7) $9876543210 - 1234567890 \times 8$
(8) $387640 - 28402 \times 4$
(9) $7628775 - 325755 \times 5$
(10) $705307060 - 38042016 \times 6$
(11) $494821309 - 43756801 \times 9$
(12) $980598 - 142857 \times 6$

13. Divide 478143296 by 26478.

Full form.

```
26478) 478143296 (18058
       26478
       213363
       211824
         153929
         132390
          215396
          211824
            3572
```

Contracted form.

```
26478) 478143296 (18058
          213363
            153929
             215396
               3572
```

Wording: 1st remainder—8 *and* **6′**, 14, carry 1; 8 *and* **3′**, 11, carry 1; 5 *and* **3′**, 8; 6 *and* **1′**, 7; 2 *and* **2′**, 4. 2nd remainder—64 *and* **9′**, 73, carry 7; 56, 63, *and* **3′**, 66, carry 6; 32, 38, *and* **5′**, 43, carry 4; 48, 52, *and* **1′**, 53, carry 5; 16, 21. 3rd remainder—40 *and* **9′**, 49, carry 4; 35, 39, *and* **3′**, 42, carry 4; 20, 24, *and* **5′**, 29, carry 2; 30, 32, *and* **1′**, 33, carry 3; 10, 13, *and* **2′**, 15. 4th remainder—64 *and* **2′**, 66, carry 6; 56, 62, *and* **7′**, 69, carry 6; 32, 38, *and* **5′**, 43, carry 4; 48, 52, *and* **3′**, 55, carry 5; 16, 21.

At this stage, if not sooner, the student will be able to guess at each figure of the quotient by mere inspection of the figures of highest denomination in the divisor and the successive dividends *without a table.* The first figure 1 is obvious, for 2 is contained twice in 4, 26 is contained only once in 47. For the second figure, 26 in 213, 8 times and so on.

Divide £19043. 13*s.* $5\frac{3}{4}$*d.* by 415.

Contracted form.

```
       £.     s.  d.   £.   s.  d.
415) 19043   13   5¾ (45   17   9
      2443
       368
      ----
       737
      3223
       318
      ----
      3821
        86
      ----
       347 farthings over.
```

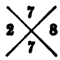
Exercise XXXV.

(1) 3097612 ÷ 8415
(2) 15000069 ÷ 9648
(3) £58943. 15*s.* $7\frac{3}{4}$*d.* ÷ 538
(4) £73914. 10*s.* $8\frac{1}{2}$*d.* ÷ 2047
(5) 328016000 ÷ 20849
(6) 495627844 ÷ 16759
(7) £438629. 12*s.* $9\frac{1}{2}$*d.* ÷ 60043
(8) 7012017 ÷ 638
(9) 35105090 ÷ 27643
(10) 459643729 ÷ 390458
(11) 326667052 ÷ 72046
(12) 2150842030246 ÷ 537689
(13) £457963824. 17*s.* $3\frac{1}{4}$*d.* ÷ 67902
(14) 1000000000 ÷ 53928
(15) 1000000 ÷ 13
(16) 15263318832 ÷ 2616
(17) 15263318832 ÷ 5834602
(18) £41265. 1*s.* $1\frac{1}{2}$*d.* ÷ 5049
(19) £509446. 8*s.* 9*d.* ÷ 10010
(20) 2911902 ÷ 178

14. Multiply 358967 by 998.

Full form.

```
 358967
    998
-------
2871736
3230703
3230703
---------
358249066
```



1st contraction.

$358967 \times 998 = 358967 \times 1000 - 358967 \times 2$

```
358967000
   717934
---------
358249066
```

2nd contraction.

```
358967... × 998
---------
358249066
```

Wording: 14 *and* 6′, 20, carry 2; 12, 14, *and* 6′, 20, carry 2′; 18, 20, *and* 0′, 20, carry 2; 16, 18, *and* 9′, 27, carry 2; 10, 12, *and* 4′, 16, carry 1; 6, 7, *and* 2′, 9; 8′; 5′; 3′.

N.B. Special care must be taken in all these contractions to work with neatness, placing the results under the corresponding figures of the subtrahend.

Exercise XXXVI.

(1) 378645916×97

(2) 14790804×996

(3) 3017682×9993

(4) 246060×95

(5) 604580900×9200

(6) 748900×910

(7) $208712 \times 93 \times 95$

(8) 267812479×9999

15. Divide 848008 by 56. According to the two interpretations of Division given in Chapter VIII. § 1, this may mean, either (*a*) "How many times is 56 contained in 848008?" or (*b*) "Distribute 848008 into 56 equal parts."

(*a*) Since $56 = 7 \times 8 = 8 \times 7$, and we have to group the 848008 things (say marbles) into lots of 56 marbles, we may first group them into lots of 8 marbles, and gather these lots 7 at a time, or first group them into lots of 7 marbles, and gather these lots 8 at a time.

```
8)848008  marbles
7)106001  lots of 8 marbles
   15143  lots of 7 × 8 or 56 marbles.

7)848008  marbles
8)121144  lots of 7 marbles
   15143  lots of 8 × 7 or 56 marbles.
```

(*b*) We have to distribute 848008 marbles among 56 persons, or among 7 companies of 8 persons, or among 8 companies of 7 persons.

```
7)848008  marbles
8)121144  marbles to each company of 8 persons
   15143  marbles to each of these persons.

8)848008  marbles
7)106001  marbles to each company of 7 persons
   15143  marbles to each person.
```

Divide 45648 by 36. $36 = 6 \times 6 = 4 \times 9 = 9 \times 4 = 3 \times 12 = 12 \times 3$.

```
6)45648     9)45648     4)45648     12)45648     3)45648
 6)7608      4)5072     9)11412       3)3804    12)15216
   1268        1268        1268         1268        1268
```

$36 = 2 \times 18$, but this resolution of 36 into factors is not available, because we only know the multiplication table up to twelves. We have thus the choice of the five different ways here given.

Divide £5159. 1*s.* 3*d.* among 45 persons.

```
    £.   s.  d.                       £.   s.  d.
5)5159   1   3                    9)5159   1   3
9)1031  16   3 to each 9           5)573   4   7 to each 5
   114  12  11 to each 1             114  12  11 to each 1
```

Exercise XXXVII.

(1) $261912 \div 56$
(2) £11537. 6*s.* 4¼*d.* $\div$ 25
(3) $205515 \div 45$
(4) £901. 1*s.* $\div$ 36
(5) $76624 \div 16$
(6) £1270. 10*s.* $\div$ 32
(7) $593075 \div 35$
(8) £62578. 9*s.* 9¾*d.* $\div$ 81
(9) $512784 \div 144$
(10) £17075. 11*s.* 2¼*d.* $\div$ 63
(11) $7777728 \div 63$
(12) £16967. 14*s.* $\div$ 144

N.B. In this Exercise the interpretation of each line must be given.

16. Divide 438000 by 10. This means, How many times is 10 contained in 438000? or, How many tens are there in 438000? *Ans.* 43800. (See Ch. I. § 11.)

Divide 3745916 by 100. *Ans.* 37459 and 16 over.

Divide 42097412 by 10000. *Ans.* 4209 and 7412 over; hence,

Learn by heart: Rule:—*To divide by any power of* 10, *strike off from the right of the dividend as many figures as there are ciphers in the divisor.* This is evidently the converse of the rule in Ch. VI. § 5.

Divide £23519. 12*s.* 8½*d.* by 10. Dividing the pounds, we obtain £2351 and £9 over; £9 and 12 shillings = 192 shillings, giving 19 shillings and 2 shillings over; 2 shillings and 8 pence = 32 pence, giving 3 pence and 2 pence over, which with the halfpenny make 10 farthings, giving 1 farthing.

	£.	s.	d.
10)	23519	12	8½
	2351	19	3¼

Divide £746915. 6*s.* 2½*d.* by 1000.

	£.	s.	d.
1000)	746(915	6	2½
	18(306		
	3(674		
	2(698		

Ans. £746. 18*s.* 3½*d.* and 698 farthings over.

N.B. We forbear giving the name to the result, as the "numerical values of the quotient and remainder are the same" under each interpretation. (See Ch. VIII. § 6.)

Exercise XXXVIII.

(1) 3257968 ÷ 10, 100, 1000, 10000.

(2) £8684. 7*s.* 6*d.* ÷ 10, 100, 1000, 10000.

(3) £39791. 13*s.* 4*d.* by 10, 100, 1000, 10000, 100000.

(4) £243519. 16*s.* 5*d.* ÷ 1000.

(5) £4000019. 6*s.* ÷ 10000.

(6) £59. 9*s.* 7*d.* ÷ 100.

17. Divide 4321232 by 56.

Under interpretation (*a*).

```
8)4321232
 7)540154 eights
    77164 fifty-sixes and 6 eights over.
```

Remainder, $6 \times 8 = 48$.

or,

```
7)4321232
 8)617318 sevens and 6 units over
    77164 fifty-sixes and 6 sevens over.
```

Remainder, $6 \times 7 + 6 = 48$.

Under interpretation (*b*).

```
8)4321232
 7)540154 to each of the 8 groups
    77164 to each one, and 6 over from each group.
```

Remainder, $6 \times 8 = 48$.

```
7)4321232
 8)617318 to each of the 7 groups, and 6 over
    77164 to each one, and 6 over from each group.
```

Remainder, $6 \times 7 + 6 = 48$.

£37591. 15*s.* 7¼*d.* ÷ 45.

```
5)37591 15  7¼
 9)7518  7  1¼ to each of the 5 sets, and 4 farthings over
    835  7  5¼ to each one, and 8 farthings over from each set.
```

Total remainder, $5 \times 8 + 4 = 44$ farthings.

or,

```
9)37591 15  7¼
 5)4176 17  3¼ to each of the 9 groups, and 8 farthings over
    835  7  5¼ and 4 farthings over from each group.
```

Total remainder, $4 \times 9 + 8 = 44$ farthings.

Exercise XXXIX.

(1) 458239 ÷ 14

(2) £670138. 12*s.* 11*d.* ÷ 66

(3) 826549 ÷ 45

(4) £764. 18*s.* 2¾*d.* ÷ 72

(5) 2761324 ÷ 81

(6) £5374. 17*s.* 6*d.* ÷ 15

(7) 14685999 ÷ 49

(8) £27632. 4*s.* 3½*d.* ÷ 64

(9) 7632419 ÷ 77

(10) £1000000 ÷ 84

(11) 5768341 ÷ 132

(12) £16349. 12*s.* 7*d.* ÷ 42

18. 374916 ÷ 70.

```
10)37491(6
    7)37491  and 6 over
       5355  and 6 × 10 + 6 = 66 over.
```

or, in one line,

```
70)37491(6
      5355  and 66 over.
```

437492563 ÷ 8000.

```
8000)437492(563
       54686  and 4563 over.
```

58317409 ÷ 359000.

```
359000)58317(409 (162
         2241
           877
           159409        Ans. 162, and 159409 over.
```

Exercise XL.

(1) 315672 ÷ 20
(2) 8409136 ÷ 30
(3) 8409136 ÷ 40
(4) 8409136 ÷ 50
(5) 437589601 ÷ 90
(6) 8392 ÷ 60
(7) 595536900 ÷ 70
(8) 2359360 ÷ 80
(9) 904813 ÷ 600
(10) 5897343 ÷ 5000
(11) 388493200 ÷ 9000000
(12) 3596250000 ÷ 5000
(13) 57892517 ÷ 63200
(14) 34603421 ÷ 7410000
(15) 368254000 ÷ 5300000
(16) 1498632000 ÷ 730
(17) 7765450000 ÷ 38500
(18) 36932215800000 ÷ 738600
(19) 487563625 ÷ 50
(20) 487563625 ÷ 500
(21) 487563625 ÷ 5000
(22) 487563625 ÷ 50000
(23) 4875636250 ÷ 50000
(24) 48756362500 ÷ 50000
(25) 487563625000 ÷ 50000

19.
5 = 10 ÷ 2
25 = 100 ÷ 4
75 = 300 ÷ 4
125 = 1000 ÷ 8
375 = 3000 ÷ 8
625 = 5000 ÷ 8
875 = 7000 ÷ 8

Hence to multiply by 5, we may multiply by 10 and divide by 2.

„	25,	„	100	„	4.
„	75,	„	300	„	4.
„	125,	„	1000	„	8.
„	375,	„	3000	„	8.
„	625,	„	5000	„	8.
„	875,	„	7000	„	8.

Multiply 437911 by 25.

```
4)43791100
  10947775        Ans. 10947775.
```

Multiply 8923761 by 375.

```
   8923761
         3
8)26771283000
  3346410375      Ans. 3346410375.
```

Exercise XLI.

(1) 381961 × 25
(2) 621852 × 75
(3) 776877 × 125
(4) 492743 × 375
(5) 276768 × 625
(6) 512634 × 875
(7) 85059 × 125
(8) 52530 × 25
(9) 385149 × 625
(10) 103416 × 75
(11) 94238 × 375
(12) 723498 × 875
(13) 427 × 25 × 25 × 25
(14) 1853 × 125 × 25 × 875
(15) 1866 × 375 × 5 × 75 × 1000
(16) 512 × 125 × 375 × 625 × 875

CHAPTER X.

SCALES OF NOTATION.

1. A Scale of Notation may be simple or compound.

In a simple scale, such as those named in Ch. I. § 10, we pass from one column to the next higher one by the *same* multiplier or *radix*. Thus in the Binary scale the radix is 2, in the Ternary 3, in the Decimal 10, and so on.

In a compound scale the radix changes; thus in the Money scale it is successively 4, 12, 10, 2, and then permanently 10. In Avoirdupois Weight it is 16, 16, 28, 4, 20, and again permanently 10, and so on.

2. Simple Scales. Addition.

Add the following quantities, expressed in the quinary scale: 2041, 41012, 12340, 123, 333, 33404.

N.B. 5×5 is written 5^2, $5 \times 5 \times 5$ is written 5^3, $5 \times 5 \times 5 \times 5$ is written 5^4, and so on.

5^4	5^3	5^2	5	1	
	2	0	4	1	3 *
4	1	0	1	2	0
1	2	3	4	0	2
		1	2	3	2
		3	3	3	1
3	3	4	0	4	2
2 0	0	4	1	3	2′

Since 5 in any column gives 1 in the column next to the left, we add each column separately, carrying every 5 as 1 to the next column, beginning (as in ordinary addition) on the right. (Ch. III. § 4.)

Wording: 4, 7, 10, 12, 13, 3′, carry 2; 5, 7, 11, 12, 16, 1′, carry 3; 7, 10, 11, 14, 4′, carry 2; 5, 7, 8, 10, 0′, carry 2; 5, 6, 10, 0′; 2′. *Answer.* 200413. (This must be read, "two, nought, nought, four, one, three.")

Exercise XLII.

(1) Add in the binary scale, 111, 1100, 1101, 1110, 11001, 1001, 11101.

(2) In the ternary scale, 1220, 2012, 2111, 210, 12112, 222, 1221.

(3) In the quaternary scale, 1032, 1222, 22321, 1211, 1002, 12223, 3232.

(4) In the quinary scale, 2341, 1234, 110, 2323, 443322, 12340, 342103.

(5) In the senary scale, 54320, 234, 5030, 24110, 25142, 33445, 55443.

(6) In the septenary scale, 6543, 6321, 1324, 235, 35264, 10235.

(7) In the octonary scale, 76321, 4623, 5276, 35402, 70607, 42354.

(8) In the nonary scale, 1235, 7834, 72684, 503785, 123456, 78064.

(9) In the undecimal scale (using *t* for ten), 6*t*432, 12579, 708*t*4, 5*tt*37*t*.

(10) In the duodecimal scale (using *t* for ten and *e* for eleven) 6*te*4*e* 7*eet*4, 897*e*5, 3364*t*, *tetet*, *eeeee*.

* Casting out fours, 4 being 1 less than the radix, just as 9 is 1 less than the radix in the decimal scale. (Cf. Ch. XI. § 10.)

(11) In each of the preceding scales, 11111 + 10101 + 11001 + 10011 + 11001 + 10111 + 1111 + 1001 + 11001 + 1 + 111 + 101.

3. Subtraction.

In the senary scale, take 45305 from 305213.

$$\begin{array}{cccccc|c}
6^5 & 6^4 & 6^3 & 6^2 & 6 & 1 & \\
\hline
3 & 0 & 5 & 2 & 1 & 3 & 4 \\
 & 4 & 5 & 3 & 0 & 5 & 2 \\
\hline
2 & 1 & 5 & 5 & 0 & 4 & 2'
\end{array}$$

Here if any figure of the subtrahend is greater than the figure of the minuend above it, we add 6 to the figure of the minuend, carrying 1 to the next column to the left. (Ch. IX. p. 102.)

Wording: 5 and $4'$ is 9, carry 1; 1 and $0'$ is 1; 3 and $5'$ is 8, carry 1; 6 and $5'$ is 11, carry 1; 5 and $1'$ is 6, carry 1; 1 and $2'$ is 3.

EXERCISE XLIII.

(1) Take in the binary scale, 101010 from 1110101.

(2) In the ternary scale, 211021 from 1002101.

(3) In the quaternary scale, 121323 from 303030.

(4) In the quinary scale, 32402 from 40000.

(5) In the senary scale, 12345 from 323520.

(6) In the septenary scale, 135066 from 423450.

(7) In the octonary scale, 3407427 from 7262520.

(8) In the nonary scale, 326784 from 2233441.

(9) In the undecimal scale, 7*t*4*t*3 from 102030.

(10) In the duodecimal scale, 7*teet*407 from 89*t*4007*te*.

(11) In each of the preceding scales, 101101011 from 110010010.

4. Multiplication.

42345 × 3 in the senary scale.

$$\begin{array}{cccccc}
6^5 & 6^4 & 6^3 & 6^2 & 6 & 1 \\
\hline
 & 4 & 2 & 3 & 4 & 5 \\
 & & & & & 3 \\
\hline
2 & 1 & 1 & 5 & 2 & 3
\end{array}$$

Wording: 15, $3'$, carry 2; 12, 14, $2'$, carry 2; 9, 11, $5'$, carry 1; 6, 7, $1'$, carry 1; 12, 13, $1'$; $2'$.

Ans. 211523.

64325 × seven, in the septenary scale.

7^4	7^3	7^2	7	1	
	6	4	3	2	5
					seven = 10
6	4	3	2	5	0

Wording: 35, 0′, carry 5; 14, 19, 5′, carry 2; 21, 23, 2′, carry 3; 28, 31, 3′, carry 4; 42, 46, 4′; 6′.

Comparing the product with the multiplicand, we see that a cipher has been added in the units' place, the other figures remaining unchanged. This might have been anticipated, for the cipher in the units' place moves each figure one step higher in the septenary scale. (Cf. Ch. V. p. 55.)

Note that in any scale the symbol for the radix is 10, and for the successive powers of the radix 100, 1000, &c.

Generally: To multiply in any scale by a power of the radix, put on as many ciphers to the right of the multiplicand as there are ciphers in the multiplier expressed in that scale. (Cf. Ch. VI. p. 67.)

462037 × 2056 in the octonary scale.

Casting out sevens:

```
 \ 6 /
1  X  6
 / 6 \
```

8^9	8^8	8^7	8^6	8^5	8^4	8^3	8^2	8	1
				4	6	2	0	3	7
						2	0	5	6
			3	4	5	4	2	7	2
		2	7	7	2	2	3	3	
1	1	4	4	0	7	6			
1	1	7	7	4	7	4	6	2	2

Ans. 1177474622.

*te*3*t*7 × *t*3*e*8 in the duodecimal scale.

Casting out elevens:

```
 \ 3 /
8  X  t
 / 3 \
```

12^8	12^7	12^6	12^5	12^4	12^3	12^2	12	1
				t	*e*	3	*t*	7
					t	3	*e*	8
			7	3	6	7	0	8
		t	0	4	6	8	5	
	2	8	9	*e*	7	9		
9	1	5	2	9	*t*			
9	5	0	8	5	7	0	5	8

Ans. 950857058.

Exercise XLIV.

(1) 1011101 × 10011 in the binary scale.
(2) 2122002 × 1212 in the ternary scale.
(3) 3130321 × 30012 in the quaternary scale.
(4) 123404 × 3204 in the quinary scale.
(5) 53210 × 1305 in the senary scale.
(6) 130666 × 651 in the septenary scale.
(7) 46734 × 730 in the octonary scale.
(8) 80600 × 71000 in the nonary scale.
(9) 457*t* × 4*tt*1 in the undecimal scale.
(10) 6*et*1 × 6*et*1 in the duodecimal scale.
(11) 1100 × 1000 in every scale.

5. Division.

7341062 ÷ 5 in the octonary scale.

$$\begin{array}{r} 5)\underline{7341062} \\ 1371643\text{—}3 \end{array}$$

Wording: 5 in 7, 1′, carry 2; (2 × 8 + 3 = 19) in 19, 3′, carry 4, 32; in 36, 7′, carry 1, 8; in 9, 1′, carry 4, 32; in 32, 6′, carry 2, 16; in 22, 4′, carry 2, 16; in 18, 3′, and 3 over.

Note that to divide by a power of the radix, we cut off as many figures from the right of the dividend as there are ciphers in the divisor, expressed in that scale.

Exercise XLV.

(1) 1000110 ÷ 1000 in the binary scale.
(2) 12112 ÷ 2 in the ternary scale.
(3) 321232 ÷ 3 in the quaternary scale.
(4) 100342 ÷ 3 in the quinary scale.
(5) 132432 ÷ 7 in the quinary scale.
(6) 543212 ÷ 4 in the senary scale.
(7) 5372461 ÷ 7 in the octonary scale.
(8) 33776426 ÷ 6 in the nonary scale.
(9) 123456 ÷ *t* in the undecimal scale.
(10) 5732*te*2 ÷ *t* in the duodecimal scale.

N.B. These scales are of little practical utility, and we therefore omit Long Division.

6. Interconversion of simple scales.

Express 437 (decimal) in the quinary scale.

$$\begin{array}{l} \quad\ \text{C. X. I.} \\ 5\underline{)4\ 3\ 7} \\ \quad 5\underline{)8\ 7} \text{ fives and 2 units} \\ \quad 5\underline{)1\ 7} \text{ twenty-fives and 2 fives} \\ \qquad\ 3 \text{ hundred-and-twenty-fives and 2 twenty-fives} \end{array}$$

Hence 437 (decimal) = 3222 (quinary).

Convert 73421 from the octonary to the ternary scale.

$$\begin{array}{l} \quad\ \underline{8^4\,8^3\,8^2\,8\ 1} \\ 3\underline{)7\ 3\ 4\ 2\ 1} \\ \quad\ 2\ 3\ 6\ 6\ 0 \text{ and 1 unit over} \end{array}$$

∴ 73421 in the octonary scale $= 23660 \times 3$ still expressed in the octonary scale and 1 unit over.

$$\begin{array}{l} 3\underline{)23660} \\ \quad 6472 \text{ and 2 over} \end{array}$$

∴ 73421 in the octonary scale $= 6472 \times 3^2 + 2 \times 3 + 1$.

$$\begin{array}{l} 3\underline{)6472} \\ \quad 2150 \text{ and 2 over} \end{array}$$

∴ $73421 = 2150 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1$.

$$\begin{array}{l} 3\underline{)2150} \\ \quad 570 \text{ and 0 over} \end{array}$$

∴ $73421 = 570 \times 3^4 + 0 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1$.

$$\begin{array}{l} 3\underline{)570} \\ \quad 175 \text{ and 1 over} \end{array}$$

∴ $73421 = 175 \times 3^5 + 1 \times 3^4 + 0 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1$.

$$\begin{array}{l} 3\underline{)175} \\ \quad 51 \text{ and 2 over} \end{array}$$

∴ $73421 = 51 \times 3^6 + 2 \times 3^5 + 1 \times 3^4 + 0 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1$.

$$\begin{array}{l} 3\underline{)51} \\ \quad 15 \text{ and 2 over} \end{array}$$

∴ $73421 = 15 \times 3^7 + 2 \times 3^6 + 2 \times 3^5 + 1 \times 3^4 + 0 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1$.

$$\begin{array}{l} 3\underline{)15} \\ \quad 4 \text{ and 1 over} \end{array}$$

∴ $73421 = 4 \times 3^8 + 1 \times 3^7 + 2 \times 3^6 + 2 \times 3^5 + 1 \times 3^4 + 0 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1$.

3)4
1 and 1 over

$\therefore 73421 = 1 \times 3^9 + 1 \times 3^8 + 1 \times 3^7 + 2 \times 3^6 + 2 \times 3^5 + 1 \times 3^4 + 0 \times 3^3 + 2 \times 3^2 + 2 \times 3 + 1.$

$$\therefore \frac{8^4\ 8^3\ 8^2\ 8\ \ 1}{7\ \ 3\ \ 4\ \ 2\ \ 1} = \frac{3^9\ 3^8\ 3^7\ 3^6\ 3^5\ 3^4\ 3^3\ 3^2\ 3\ \ 1}{1\ \ 1\ \ 1\ \ 2\ \ 2\ \ 1\ \ 0\ \ 2\ \ 2\ \ 1}$$

Mod. op.:

3)73421
3)23660—1
3)6472—2
3)2150—2
3)570—0
3)175—1
3)51—2
3)15—2
3)4—1
1—1

Ans. 1112210221.

In reducing to the decimal scale, the following method may be used with equal advantage:

Reduce 462351 from the septenary to the decimal scale.

$7^5\ 7^4\ 7^3\ 7^2\ 7\ \ 1$
4 6 2 3 5 1
7
(thirty-four) 34 (7^4)
7
(two hundred & forty) 240 (7^3)
7
&c. 1683 (7^2)
7
11786 (7)
7
82503 units

or, as before:

$7^5\ 7^4\ 7^3\ 7^2\ 7\ \ 1$
ten) 4 6 2 3 5 1
ten) 3 3 0 2 4—3
ten) 2 2 5 6—0
ten) 1 4 5—5
ten) 1 1—2
0—8

Ans. 82503.

4 of the 6th column gives $7 \times 4 = 28$ of the 5th column, which, with the 6 already in that column, gives 34. 34 of the 5th column gives 7×34 of the 4th column, and adding in the 2 gives 240, &c.

EXERCISE XLVI.

(1) Express 1024* in the binary scale.

(2) Express 719 in the binary scale.

(3) Express 719 in the ternary scale.

(4) Express 1000 in the quaternary scale.

(5) Express 760 in each scale from the binary to the duodecimal scale.

(6) Express 20453 in the same scales.

(7) Convert 3742 (nonary) to the decimal scale.

(8) Convert 51342 (senary) to the decimal scale.

(9) 123402 (quinary) to the decimal scale.

(10) 45312 (septenary) to the decimal scale.

(11) 10111001 (binary) to the decimal scale.

(12) 4*eete*3 (duodecimal) to the decimal scale.

(13) Convert 11001 (binary) to the quinary scale.

(14) 31426 (octonary) to the septenary scale.

(15) 12121210 (ternary) to the quaternary scale.

(16) 5*t*31*t*9 (undecimal) to the nonary scale.

(17) *te*3418 (duodecimal) to the octonary scale.

(18) 100000000 (binary) to each of the other scales.

7. COMPOUND SCALES. These methods are applicable to the interconversion of £. *s. d.*, weights and measures of length, capacity, time, &c. We proceed to give examples of the leading weights and measures, but propose to enter more fully into the whole subject at a later stage.

Reduce 7419 farthings to £. *s. d.*

$$\begin{array}{r|l} 4\,)\,\underline{7419} & \text{farthings} \\ 12\,)\,\underline{1854} & \text{pence and 3 farthings} \\ 20\,)\,\underline{154} & \text{shillings and 6 pence} \\ 7 & \text{pounds and 14 shillings.} \end{array}$$

Ans. £7. 14*s.* 6¾*d.*

EXERCISE XLVII.

(1) Reduce 1000000 farthings to £. *s. d.*

(2) Find the value of a million penny postage-stamps.

(3) Reduce 987654321 farthings to £. *s. d.*

* Where no scale is specified, the decimal scale is understood.

(4) Reduce 2479 sixpences to £. *s.* *d.*

(5) Reduce 2573 half-crowns to £. *s.* *d.* (N.B. 8 half-crowns = £1).

(6) Reduce 17019 fourpenny-pieces to £. *s.* *d.*

(7) Reduce to £. *s.* *d.* the following:

1. 43794 farthings.	4. 111840 farthings.
2. 47901 farthings.	5. 197311 halfpence.
3. 637425 farthings.	6. 16049 pence.

8. Lengths.

12 inches (in.)	=	1 foot (ft.)
3 feet	=	1 yard (yd.)
220 yards	=	1 furlong (fur.)
8 furlongs (1760 yards)	=	1 mile (m.)

Reduce 37989511 inches to miles, &c.

$$\begin{array}{rll} & 12)\underline{37989511} & \text{inches} \\ & 3)\underline{3165792} & \text{feet and 7 inches} \\ 220\left\{\begin{array}{l} \\ \\ \end{array}\right. & \begin{array}{r} 20)\underline{105526(4} \\ 11)\underline{52763} \end{array} & \begin{array}{l} \text{yards and 0 feet} \\ \text{scores of yards and 4 yards} \end{array} \\ & 8)\underline{4796} & \text{furlongs and 7 scores of yards} \left.\begin{array}{l} \\ \end{array}\right\} \begin{array}{l} 7 \times 20 + 4 \\ = 144 \text{ yards} \end{array} \\ & 599 & \text{miles and 4 furlongs.} \end{array}$$

Ans. 599 miles, 4 furlongs, 144 yards (0 feet), 7 inches.

EXERCISE XLVIII.

(1) Reduce 79 inches to feet.

(2) „ 159 inches to yards, &c.

(3) „ 1000 inches to fathoms (1 fathom = 6 feet).

(4) „ 5000 yards to miles.

(5) 1. Reduce 5317 inches to yards.
2. „ 16029 „ „
3. „ 867 „ „
4. „ 1868 „ „
5. „ 4428 „ „
6. „ 2340 „ „

(6) 1. Reduce 176000 yards to miles.
2. „ 1000000 „ „
3. „ 74912 „ „
4. „ 16016 „ „

(7) Reduce 500497056 inches to miles, &c.

9. 100 links = 1 chain (22 yards)
80 chains = 1 mile

EXERCISE XLIX.

(1) Reduce 7843 links to chains.
(2) „ 53984 links to miles.
(3) „ 174986 links to miles.
(4) „ 1000000 links to miles.

10. 4 nails (n.) = 1 quarter
4 quarters = 1 yard

EXERCISE L.

(1) Reduce 243 nails to yards.
(2) „ 587 „ „
(3) „ 1024 „ „

11. Liquid Measure.

4 gills = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gn.)

EXERCISE LI.

(1) Reduce 5317 pints to gallons.
(2) „ 20000 gills to gallons.
(3) „ 3719 quarts to gallons.

12. Dry Measure.

2 gallons = 1 peck (pk.)
4 pecks = 1 bushel (bus.)
8 bushels = 1 quarter (qr.)

EXERCISE LII.

(1) Reduce 1700 gallons to quarters.
(2) „ 1359 gallons to quarters.
(3) „ 1000000 gallons to quarters.

13. Paper Measure.

24 sheets = 1 quire
20 quires = 1 ream

EXERCISE LIII.

(1) Reduce 1750 sheets to reams.
(2) „ 157 quires to reams.
(3) „ 1920 sheets to reams.

14. Weights.

1. *Avoirdupois.*

16 drams (dr.)	=	1 ounce (oz.)
16 ounces	=	1 pound (lb.)
28 lbs.	=	1 quarter (qr.)
4 qrs. (112 lbs.)	=	1 hundred-weight (cwt.)
20 cwts.	=	1 ton.

Exercise LIV.

Reduce :

(1) 1250 drams to ounces.
(2) 512 drams to ounces.
(3) 10000 drams to lbs.
(4) 2055 drams to lbs.
(5) 5040 lbs. to cwts.

Reduce :

(6) 11111 lbs. to cwts.
(7) 100000 lbs. to tons.
(8) 25200 lbs. to tons.
(9) 1000 cwts. to tons.
(10) 27419 lbs. to tons.

2. *Troy.*

15.

24 grains (gr.)	=	1 penny-weight (dwt.)
20 dwts.	=	1 ounce (oz.)
12 oz.	=	1 lb.

Exercise LV.

(1) Reduce 500 grains to dwts.
(2) „ 5760 grains to lbs.
(3) „ 7000 grains to lbs.
(4) „ 157409 grains to lbs.

N.B. Compare the results of (2) and (3). 7000 grains = 1 lb. av., which is therefore greater than 1 lb. troy. 1 oz. av., however, is less than 1 oz. troy.

16. Reduce 45 miles to feet.

$$\begin{array}{rrl} & 45 & \text{miles} \\ & 8 & \\ \hline & 360 & \text{furlongs} \\ \multirow{3}{*}{220} & 11 & \\ \cline{2-2} & 3960 & \\ & 20 & \\ \hline & 79200 & \text{yards} \\ & 3 & \\ \hline & 237600 & \text{feet.} \end{array}$$

Ans. 237600 ft.

Reduce 5 miles, 3 fur., 143 yds., 1 ft., to inches.

$$\begin{array}{r l}
5 & \text{m., 3 fur., 143 yds., 1 ft.} \\
\underline{8} & \\
43 & \text{fur.} \\
\underline{11} & \\
473 & \\
\underline{20} & \\
9603 & \text{yds.} \\
\underline{3} & \\
28810 & \text{ft.} \\
\underline{12} & \\
345720 & \text{inches.} \qquad \textit{Ans.}\ 345720 \text{ in.}
\end{array}$$

Reduce 4 tons, 13 cwt., 1 qr., 11 oz., to ounces.

$$\begin{array}{r l}
4 & \text{tons, 13 cwt., 1 qr. (0 lbs.), 11 oz.} \\
\underline{20} & \\
93 & \text{cwt.} \\
\underline{4} & \\
373 & \text{qrs.} \\
\underline{4} & \\
1492 & \\
\underline{7} & \\
10444 & \text{lbs.} \\
\underline{16} & \\
167115 & \text{ounces.} \qquad \textit{Ans.}\ 167115 \text{ oz.}
\end{array}$$

EXERCISE LVI.

(1) Reduce 7 yds., 2 ft., 9 in., to inches.
(2) „ 3 m., 1 fur., 93 yds., to feet.
(3) „ £73. 14*s.* 7¼*d.* to farthings.
(4) „ £100 to sixpences.
(5) „ 6*s.* 8*d.* to fourpenny-pieces.
(6) „ 5 half-crowns to threepenny-pieces.
(7) „ £7. 17*s.* 6*d.* to sixpences.
(8) „ £99. 19*s.* 11¾*d.* to farthings.
(9) „ £19. 15*s.* to crowns.
(10) „ £52. 10*s.* to half-crowns.
(11) „ £6. 13*s.* 7½*d.* to halfpence.
(12) „ £18. 2*s.* 9½*d.* to farthings.
(13) „ 100 tons to lbs.

(14) Reduce 8 cwt., 11 oz., to oz.
(15) „ 5 reams to sheets.
(16) „ 1 year to seconds.
(17) How many beats does the seconds' pendulum make in a week?
(18) How many miles can I travel for £3. 7*s.* 10*d.* at a penny a mile?

17. Addition, Subtraction, Multiplication and Division of Weights and Measures may be performed by methods similar to those already given for £. *s.* *d.*

(1) Add the following: 17 tons, 5 cwt., 1 qr., 13 lbs.; 25 tons, 3 cwt., 15 lbs.; 9 tons, 6 cwt., 1 qr., 21 lbs.; 14 cwt., 3 qrs., 11 lbs.; 73 tons, 17 lbs.; 1 ton, 13 cwt., 14 lbs.

Tons.	cwt.	qrs.	lbs.
17	5	1	13
25	3	0	15
9	6	1	21
	14	3	11
78	0	0	17
1	13	0	14
127	3	0	7

Wording: 4, 11, 12, 13, 18, 21, carry 2; 3, 4, 5, 7, 8, 9, 91 lbs. (28 in 91, 3 and 7 over), 7′, carry 3; 6, 7, 8, 0′, carry 2; 5, 9, 15, 18, 23′, carry 2; 3, 4, carry 2; 3, 6, 15, 20, 27′, carry 2; 9, 11, 12′.

Ans. 127 tons, 3 cwt., 7 lbs.

(2) From 19 tons, 5 cwt., 11 lbs., take 2 tons, 17 cwt., 1 qr., 27 lbs.

Tons.	cwt.	qrs.	lbs.
19	5	0	11
2	17	1	27
16	7	2	12

Wording: 27 and 12′ is 39 (adding 1 qr. = 28 lbs. to minuend and subtrahend), carry 1; 2 and 2′ is 4, carry 1; 18 and 7′ is 25, carry 1; 3 and 6′ is 9; 1′.

Ans. 16 tons, 7 cwt., 2 qrs., 12 lbs.

(3) 15 tons, 3 cwt., 1 qr., 13 lbs. × 4023.

	Tons.	cwt.	qrs.	lbs.
I.	15	3	1	13 × 3
X.	151	13	2	18 × 2
C.	1516	16	2	12
M.	15168	6	0	8 × 4
	60673	4	1	4
	303	7	1	8
	45	10	0	11
	61022	1	2	23

Ans. 61022 tons, 1 cwt., 2 qrs., 23 lbs.

(4) 251 tons, 3 cwt., 4 lbs. ÷ 11 tons, 5 cwt., 1 lb.

Tons.	cwt.	qrs.	lbs.	
11	5	0	1	I.
112	10	0	10	X.

					X.I.
11 tons, 5 cwt., 0 qrs., 1 lb.)	251	3	0	4	(22
	225	0	0	20	
	26	2	3	12	
	22	10	0	2	
	3	12	3	10	

Ans. 22 times and 3 tons, 12 cwt., 3 qrs., 10 lbs. over.

(5) 734 tons, 4 cwt., 1 qr., 12 lbs. ÷ 17.

	Tons.	cwt.	qrs.	lbs.	Tons.	cwt.	qrs.	lbs.
17)	734	4	1	12	(43	3	3	4
	54							
	3							
	64							
	13							
	53							
	2							
	68							

Ans. 43 tons, 3 cwt., 3 qrs., 4 lbs.

EXERCISE LVII.

(1) Add 50 tons, 17 cwt., 3 qrs., 15 lbs.; 12 tons, 12 cwt., 12 lbs.; 25 tons, 11 cwt., 7 lbs.; 33 tons, 15 cwt., 2 qrs., 23 lbs.

(2) Add 5 oz., 13 dwt., 7 grs.; 11 oz., 10 dwts., 17 grs.; 2 oz., 15 grs.; 17 dwt., 14 grs.; 3 oz., 23 grs.

(3) 5 miles, 7 fur., 13 yds. + 19 miles, 1 fur., 200 yds. + 11 miles, 3 fur., 37 yds. + 2 fur., 183 yds. + 17 miles, 5 fur., 177 yds.

(4) 24 hrs., 11 min., 25 sec. + 19 hrs., 17 min., 11 sec. + 1 hr., 49 min., 50 sec. + 20 hrs., 20 min., 20 sec.

(5) 13 tons, 17 cwt., 13 lbs. — 4 tons., 15 cwt., 3 qrs., 20 lbs.

(6) 9 oz., 3 dwt., 3 grs. — 3 oz., 13 dwt., 7 grs.

(7) 20 miles — 11 miles, 1 fur., 17 yds.

(8) From 7 yds., 11 inches, take 3 yds., 2 feet, 7 inches.

(9) 8 tons, 11 cwt., 1 qr., 14 lbs. × 308.

(10) 5 oz., 5 dwt., 17 grs. × 3160.

(11) 5 miles, 155 yds. × 188.

(12) 237 tons, 18 cwt., 3 qrs., 20 lbs. ÷ 4 cwt., 3 qrs., 1 lb.

(13) 20 lbs. troy $\div$ 15 dwts., 11 grs.

(14) 7 tons, 11 cwt., 14 lbs. $\div$ 7.

(15) 13 lbs. troy $\div$ 32.

(16) 173 tons, 5 cwt., 1 qr., 3 lbs. $\div$ 19.

(17) 24 miles $\div$ 55.

(18) 186 yds., 11 inches $\div$ 89.

(19) $360^0 \div 365$.

(20) How many copies can be printed off 13 reams, 7 quires, each consisting of 11 sheets?

CHAPTER XI.

PROPERTIES OF NUMBERS, &c.

1. WE have hitherto been considering almost exclusively those properties of numbers which belong specially to given numbers; thus, the only number which exceeds 15 by 7 is 22. But some general properties of numbers have been tacitly assumed as true. The expression, "general properties of numbers," requires elucidation. Take, for example, any two numbers, say, 40 and 17; it is evident that $40 + 17 = 17 + 40$; but as this is true of *any* two numbers, we may use more general symbols. Let l and s stand for two different numbers whose value is unknown to us; we still know that $l + s = s + l$, or, in other words, that numbers may be added in any order. This was taken for granted in our rules for addition. We further know that $(l - a) + (s + a) = l + s$, or that the sum of two numbers is not altered by transferring a portion of one of the numbers to the other. This also is accepted as true in "carrying" in addition.

If we are informed that of the two numbers l and s, l stands for the larger of the two, we know that $l - s$ is a possible, and $s - l$ an impossible, quantity. Again, we know that $(l + a) - (s + a) = l - s$, or that "the difference between two numbers will not be altered, if the same quantity be added to both of them." (Ch. IX. § 8.) Similarly we know and have assumed through Ch. V. and

VI. that $(l + s) \times m = l \times m + s \times m$, or that the sum of two (or more) numbers is multiplied by another number if each of the addenda is so multiplied; thus: $27 \times 5 = 20 \times 5 + 7 \times 5$.

We proceed to other general properties of numbers.

2. If one number can be divided by another number *without remainder*, the divisor is called a MEASURE of the dividend, and the dividend a MULTIPLE of the divisor. Thus 20 can be divided by 5 without remainder, therefore 5 is called a MEASURE of 20, and 20 a MULTIPLE of 5. A length of 20 inches can be *measured* by a rod 5 inches in length, but not by one of 6 inches. 20 can be obtained from 5 by *multiplying* it.

The measures of 6 are 6, 3, 2, 1. It is required to find the measures of 48.

48	
1)	(48
2)	(24
3)	(16
4)	(12
6)	(8

Since $1 \times 48 = 48$, 1 and 48 are measures of 48; since $2 \times 24 = 48$, 2 and 24 are measures of 48, and as there are no numbers between 1 and 2, there are no measures between 48 and 24, for any such measure would have to be contained more than once and less than twice. Next take 3; since $3 \times 16 = 48$, 3 and 16 are measures of 48, and, as before, no measure of 48 can lie between 24 and 16, there being no number between 2 and 3. Take 4; since $4 \times 12 = 48$, 4 and 12 are measures of 48. 5 is not a measure of 48. $6 \times 8 = 48$, so that 6 and 8 are measures of 48; and as we have now all measures between 1 and 6, we must have all between 48 and 8; and as 7, the only number between 6 and 8, is not a measure of 48, we have now found *all possible* measures of 48. They are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

EXERCISE LVIII.

(1) Find all measures of 10, 12, 16, 20, 36, 60, 100, 112, 120, 144, 240, 360, 960, 1000, 1760, 5760, 7000.

3. *Q.* What is the greatest measure of any number?

A. The number itself.

Q. What is the least?

A. Unity.*

Q. Find the measures of 17.

A. 1 and 17.

A number which has no measures but itself and unity is called a PRIME NUMBER; other numbers are called COMPOSITE NUMBERS.

EXERCISE LIX.

(1) Write out all the prime numbers under 100.

(2) Classify the following numbers into prime and composite numbers: 91, 111, 113, 117, 119, 121, 131, 133.

4. All the multiples of 2 are called EVEN NUMBERS; the others are ODD NUMBERS; and hence the even and the odd numbers lie alternately.

It is evident that 2 is the only prime number that is even; note, however, that though all prime numbers (except 2) are odd, not all odd numbers are prime.

(*a*) Even added to even yields even: thus, $8 + 10 = 18$, for an exact number of twos added to an exact number of twos must be an exact number of twos.

(*b*) Even added to odd yields odd: thus, $8 + 9 = 17$, for an odd number is an exact number of twos $+ 1$, and an exact number of twos added to an exact number of twos $+ 1$ must yield an exact number of twos $+ 1$, i.e. an odd number.

(*c*) Odd added to odd yields even: thus, $7 + 9 = 16$, for an exact number of twos $+ 1$, added to an exact number of twos $+ 1$, must give an exact number of twos $+ 2$, that is, altogether, an exact number of twos. Conversely,

(*d*) Even taken from even leaves even.

(*e*) Even taken from odd leaves odd.

(*f*) Odd taken from even leaves odd.

(*g*) Odd taken from odd leaves even.

* We are only considering whole numbers or integers.

(*h*) Even multiplied by even or odd yields even, for any exact number of twos multiplied by any number must be an exact number of twos.

(*i*) Odd multiplied by odd yields odd, for the multiplier is an even number + 1, so that the multiplicand has to be repeated this even number of times and once more; the result of taking it an even number of times is to give an even number, and adding to this result once the multiplicand, which is odd, the final result will be even + odd, or odd. Conversely,

(*j*) When the dividend is a multiple of the divisor, even divided by odd yields even, because even multiplied by odd yields even.

(*k*) Even divided by even may give either even or odd, because even multiplied by either yields even.

(*l*) Odd divided by odd yields odd.

(*m*) Odd cannot be divisible by even without remainder.

Examination of these rules shews that in Addition and Subtraction likes give even, unlikes odd; and that in Multiplication the product is even unless both factors are odd.

5. If one number is a measure of another number, it will also be a measure of all its multiples; thus, 7 is a measure of 14, and is therefore a measure of any number of fourteens. Generally, if a is a measure of b, it is a measure of $m \times b$.

6. If one number is a measure of two other numbers, it will also be a measure of their sum and of their difference. Thus, if there be two numbers, each of which is an exact number of [fives] (say), their sum or their difference must still be an exact number of [fives], i.e. [five] will be a measure of either their sum or their difference. Generally, if m measures a and b, it will measure $a + b$ and $a - b$. Combining this with what was said in § 5, we see that a number which is a measure of any two numbers is also a measure of the sum or difference of any of their multiples.

7. If a number measures one *only* of two numbers, it cannot measure either their sum or their difference. If one number is an exact number of [fives], and another is an exact number of [fives] + [three], their sum will be an exact number of [fives] + [three], and their

difference will be an exact number of [fives] + [three], if the multiple of [five] is the less of the two quantities; or an exact number of [fives] — [three], if the multiple of [five] is the greater of the two quantities, and this difference therefore will not be measured by [five].

If a number measures *neither* of two numbers, it may or may not measure their sum and their difference; [five] will measure the sum of two numbers, if the two remainders together make up another [five]; and their difference, if the remainders are the same in each; e.g., 5 is not a measure of 43 nor of 32, but is a measure of their sum, since 43 = a number of fives + 3, and 32 = a number of fives + 2, so that the sum is an exact number of fives + 5, i.e. is an exact number of fives. Again, 5 is not a measure of 23 nor of 63, but is a measure of their difference, for 63 is a number of fives + 3, and 23 is a number of fives + 3.

8. Divisibility of Numbers.

Examine the number 153576.

(*a*) This is 153576 units or 15357 tens and 6 units; 2 is a measure of 10, and ∴ of 15357 tens (§ 5); but 2 is also a measure of 6; ∴ it is a measure of 15357 tens + 6 units (§ 6), i.e. of 153576. The same reasoning will hold for any number; therefore the divisibility by 2 depends solely upon the figure in the units' place; for every number above ten is a number of tens + some units. The tens are always even, and if the units are also even, we have even + even, which is even (§ 4, *a*); if the units are odd, we have even + odd, which is odd (§ 4, *b*).

Learn by heart: *A number is divisible by* 2 *if the figure in the units' place is even.*

(*b*) The number 153576 is 1535 hundreds and 76 units. 4 is a measure of 100 ($25 \times 4 = 100$), and ∴ of 1535 hundreds; but 4 is also a measure of 76; ∴ it is a measure of 1535 hundreds + 76 units, i.e. of 153576. If 4 had not measured the quantity after the hundreds, it could not have measured the whole number (§ 7).

Learn by heart: *A number is divisible by* 4 *if the quantity in the units' and tens' places is divisible by* 4.

(*c*) The number 153576 is 153 thousands and 576 units. 8 is a measure of 1000 ($125 \times 8 = 1000$), and ∴ of 153 thousands; but 8 is also a measure of 576, and ∴ of 153 thousands + 576 units, i.e. of 153576. If 8 had not measured the quantity after the thousands, it could not have measured the whole number.

Learn by heart: *A number is divisible by* 8 *if the quantity in the units', tens' and hundreds' places is divisible by* 8.

N.B. 2 is the 1st, 4 the 2nd, and 8 the 3rd power of 2; and in considering the divisibility of numbers by these, we connect 2 with 1 figure, 4 with 2 figures, and 8 with 3 figures, from the right.

(*d*) Reasoning analogous to that given in (*a*) proves that *a number is divisible by* 5 *if the figure in the units' place is either* 5 *or* 0. [Let the pupil write out the proof of this.]

(*e*) $10 = 9 + 1$; $100 = 99 + 1$; $1000 = 999 + 1$, &c.; ∴ every power of 10 is an exact number of nines + 1.

Analyse 153576. It is 6 units, 7 tens, 5 hundreds, 3 thousands, 5 ten-thousands, and 1 hundred-thousand. Breaking up these different values into groups of nines and units, we obtain:

	Nines.		Units.
From the 6 units	—		6
„ 7 tens ($7 \times 9 + 7$)	7	and	7
„ 5 hundreds ($5 \times 99 + 5$)	55	„	5
„ 3 thousands ($3 \times 999 + 3$)	333	„	3
„ 5 ten-thousands ($5 \times 9999 + 5$)	5555	„	5
„ 1 hundred-thousand ($1 \times 99999 + 1$)	11111	„	1

which is an exact number of nines + ($6 + 7 + 5 + 3 + 5 + 1$) units, or an exact number of nines + 27 units; but 27 is itself a multiple of 9; ∴ the whole is divisible by 9. Note that 27 = the sum of the digits in 153576.

Learn by heart: *A number is divisible by* 9 *if the sum of its digits* is divisible by* 9.

(*f*) Reasoning analogous to that just given shews that any number is an exact number of nines + the sum of its digits. The nines must be divisible by 3; if, therefore, the sum of the digits is also divisible by 3, the whole is divisible by 3.

* The word digit is derived from the Latin *digitus*, a finger, the fingers being the natural counters.

Learn by heart: *A number is divisible by* 3 *if the sum of its digits is divisible by* 3.

NOTE. Every number divisible by 9 is divisible by 3; but not every number divisible by 3 is divisible by 9.

(*g*) Any number which is divisible by 6 will stand both the tests of 2 and 3; for sixes can be broken up into twos or into threes. Conversely, only numbers divisible by 6 can stand both these tests, for a number not divisible by 6 must be some sixes + 1, 2, 3, 4 or 5, no one of which remainders can be arranged both in twos and threes.

Learn by heart: *A number is divisible by* 6 *if it is divisible both by* 2 *and by* 3.

(*h*) $10^2 = 100 = 99 + 1 = 9 \times 11 + 1.$
$10^4 = 10000 = 9999 + 1 = 909 \times 11 + 1.$
$10^6 = 1000000 = 999999 + 1 = 90909 \times 11 + 1$, &c.,

or any even power of 10 diminished by 1 is a multiple of 11.

Take the number 1493756, which is 56 units + 37 hundreds + 49 ten-thousands + 1 million. Breaking up these different values into groups of elevens and units, we obtain:

		Elevens.		Units.
56 units	=	—		56
37 hundreds	=	37 × 9	and	37
49 ten-thousands	=	49 × 909	„	49
1 million	=	90909	„	1
				143

which is some elevens + 143 units; but 143 is itself a multiple of 11, ∴ 11 is a measure of the whole number 1493756. Hence if the sum of the successive pairs of digits be divisible by 11, the number is divisible by 11. Instead of writing the digits in pairs, it is shorter to add them up alternately, considering the digits in the odd (1st, 3rd, 5th, &c.) places as units, and those in the even places as tens; e.g. in 1493756:

Wording: 6, 13, 22, 23′, carry 2; 7, 10, 14′. *Ans.* 143.

Learn by heart: *A number is divisible by* 11 *if the sum of its digits, added up alternately, beginning at the units' place, is divisible by* 11.

(*i*) Reasoning analogous to that in (*g*) shews that a number is divisible by 12 if it will stand the tests of 3 and of 4. [Let the pupil write out the proof of this.]

[(*j*) For divisibility by 7, many tests have been suggested, but no one of them is shorter than actual trial. The pupil may exercise his ingenuity in proving the following methods:

(α) Break the given number into pairs as for 11; divide the pair of highest designation by 7, double the remainder, if any, and add this product to the next pair; divide this sum by 7, and again add the double of its remainder to the next pair, and so on; if the last remainder is 0 or a multiple of 7, the whole is so.

(β) Break into pairs as before; to the pair of lowest designation add twice the next, four times the third, once the fourth, twice the fifth, four times the sixth, &c.; if the result is divisible by 7, the number is so.

(γ) If the difference between double the units and once the tens is 0, or a multiple of 7, the number itself is a multiple of 7. For short numbers this last method is useful.]

EXERCISE LX.

Determine by inspection the measures under 13 of the following numbers: 504, 405, 315, 168, 451, 512, 98, 1080, 9999, 864, 1296, 6144, 7020, 7040, 33264, 142857, 999999, 2520.

9. CASTING OUT NINES.

We can now demonstrate the truth of the rules given above for testing the accuracy of the results in Addition, Subtraction, Multiplication and Division.

It was proved in § 8 (*e*, *f*) that "any number is an exact number of nines + the sum of its digits." Hence the remainder of a number divided by 9 may be found by casting out the nines from the sum of the digits. In the method given in Ch. II. § 4, the nines were rejected as fast as they were obtained, which is obviously the same as casting them out at the end.

Addition:

655	7
1362	3
34052	5
15	6
36084	3′

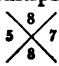
The effect of casting out nines must evidently be the same, whether we cast them out from the addenda separately or from their sum.

Subtraction.

a.	4371	6	*b.*	43726	4	*c.*	56143	1
	456	6		18237	3		12345	6
	3915	0′		25489	1′		43798	4′

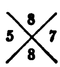
In (*a*) we take a certain number of nines + 6 from another number of nines + 6, and must therefore have an exact number of nines left. In (*b*) we take a certain number of nines + 3 from another number of nines + 4, and must therefore have an exact number of nines + 1 left. In (*c*) we take a certain number of nines + 6 from another number of nines + 1 or (as it may be called) a number of nines + 10, and we must therefore have a certain number of nines + 4 left.

Multiplication:



a. 37598×7

263186



b. 37598×457

```
   263186
  187990
 150392
 17182286
```

In (*a*) the multiplicand is a certain number of nines + 5, which, multiplied by 7, is a certain number of nines + 7 × 5 or 35. In (*b*) the same multiplicand is to be multiplied by a certain number of nines + 7; multiplication by nines yields nines, and the multiplication by 7 gives, as in (*a*), a certain number of nines + 5 × 7 or 35, and this is in each case a certain number of nines + 8, ∴ the product should be a multiple of nine + 8.

Division:

a. 3586) 168542 (47
25102
....

b. 3586) 168665 (47
25225
123

In (*a*) the divisor is contained 47 times exactly in the dividend; therefore the dividend $= 47 \times$ the divisor, and the test for multiplication applies. In (*b*) the divisor 3586 taken 47 times from the dividend leaves the remainder 123; therefore $47 \times 3586 + 123 =$ the dividend 168665, whence the truth of the rule is apparent.

[10. Divisibility of Numbers in other simple Scales of Notation. There are three obvious cases to be considered.

Case I. Where the proposed divisor is a measure of any power of the radix. If it measures the first power of the radix, the criterion of divisibility is the units' figure; if it measures the units' figure, it will measure the given number, and if not, not. (Cf. § 8, *a*, *d*.) If it measures the second power, the criterion is the last two figures; if the third power, the last three. (Cf. § 8, *b*, *c*.)

Case II. Where the proposed divisor is 1 less than the radix, or a factor of this number. The criterion is the sum of the digits. (Cf. § 8, *e*, *f*.)

Case III. Where the proposed divisor is 1 more than the radix. The criterion is the sum of the digits added up alternately, beginning at the units' place. (§ 8, *h*.)

EXERCISE LXI.

By what numbers not exceeding the radix are the following divisible?

(1) 23054, 12304, 5523 (senary).

(2) 112235, 16245, 43700 (octonary).

(3) 444444, 52836, 78780 (nonary).

(4) 7347*t*2, *tteet*, 486310 (duodecimal).]

11. RESOLUTION INTO PRIME FACTORS. Every number is either prime or is the product of two or more prime numbers or their powers. Thus 5 is prime; 6 is the product of 2 and 3, which are prime; 8 is $2 \times 2 \times 2 =$ the third power of 2; and $12 = 3 \times 2 \times 2 = 3$ times the second power of 2.

Find the prime factors of 23760. In § 8 we have given rules for determining by inspection the presence of the prime factors, 2, 3, 5, 7, 11. Applying these to the number 23760, we find that it is divisible by 2 four times successively, by 3 three times successively,

by 5 and by 11; therefore $23760 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 11$. It is convenient to write this, $2^4 \times 3^3 \times 5 \times 11$.

Mod. op:

$$\begin{array}{r} 2)\underline{23760} \\ 2)\underline{11880} \\ 2)\underline{5940} \\ 2)\underline{2970} \\ 3)\underline{1485} \\ 3)\underline{495} \\ 3)\underline{165} \\ 5)\underline{55} \\ 11 \end{array}$$

Ans. $2^4 \times 3^3 \times 5 \times 11$.

Exercise LXII.

Resolve into prime factors:

(1) 6	(5) 36	(9) 120	(13) 1320	(17) 7000
(2) 8	(6) 40	(10) 143	(14) 1760	(18) 8140
(3) 15	(7) 91	(11) 240	(15) 1845	(19) 8712
(4) 16	(8) 96	(12) 720	(16) 5760	(20) 1848

12. When all the prime factors of a number are found, all its other factors can be determined from them, since these are only products of the former. For example, $84 = 2^2 \times 3 \times 7 = 2 \times 2 \times 3 \times 7$; the only other factors of 84 are those which can be compounded from these prime factors, viz.

$2 \times 2 = 4$	$2 \times 7 = 14$	$2 \times 2 \times 3 = 12$	$2 \times 3 \times 7 = 42$
$2 \times 3 = 6$	$3 \times 7 = 21$	$2 \times 2 \times 7 = 28$	$2 \times 2 \times 3 \times 7 = 84$

Hence the factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84; as we have found in a more convenient manner in § 2.

13. In Exercise LXII., such numbers only have been chosen as have not more than one prime factor exceeding 11, and none exceeding 100; but if a number be proposed not subject to these limitations, how are we to determine its prime factors?

Find the prime factors of 867. The factor 3 is obvious: $867 = 3 \times 289$.

$$\begin{array}{r} 3)\underline{867} \\ 289 \end{array}$$

289 is not divisible by 2, 3, 5, 7 or 11, but we are not yet justi-

fied in declaring it prime, as it may have factors exceeding 11. Try 13, and we find the remainder 3.

```
13)289(22
    29
     3
```

The next prime number is 17;* on trying which, we find that it is contained exactly 17 times, $\therefore 867 = 3 \times 17^2$.

```
17)289(17
    119
```

Since $289 = 17 \times 17$, 289 can have no factor higher than 17, since such a factor would have to be contained in 289 less than 17 times, and 289 would therefore be divisible by some number less than 17, which we have found not to be the case.

Find the factors of 283. 283 is not divisible by 2, 3, 5, 7 or 11. Try 13. There is a remainder 10.

```
13)283(21        21×13=273, 22×13=286
    23
    10
```

Try 17. There is a remainder 11.

```
17)283(16        16×17=272, 17×17=289
   113
    11
```

Therefore 283 can have no factor exceeding 17, for such a factor would have to be contained less than 17 times, that is to say, 283 would be divisible by some number less than 17, which we have found not to be the case. Therefore 283 is prime. Hence, in seeking the factors, prime or composite, of any number, we need only try prime numbers until the quotient is equal to or less than the divisor. Thus, for numbers less than 100, we need only try primes under 10.

EXERCISE LXIII.

Classify the following numbers into prime and composite, and resolve each composite number into its prime factors: 101, 765, 169, 247, 2109, 365, 1367, 1867, 4019, 3059, 483, 99, 999, 9999, 99999, 999999.

* No composite number need be tried, it being "compounded" of earlier prime numbers, which have already been tried.

14. GREATEST COMMON MEASURE.

The measures of 20 and 30 respectively are: 1, 2, 4, 5, 10, 20; 1, 2, 3, 5, 6, 10, 15, 30. We see that these two numbers have the measures 1, 2, 5, 10, in common, while 4 and 20 belong only to 20; 3, 6, 15 and 30 only to 30.

Learn by heart: *The measures that two or more numbers have in common are called their* COMMON MEASURES, *and the greatest of these is called their* GREATEST COMMON MEASURE, *which is indicated by the letters* G. C. M.

15. The only common measure of 8 and 15 is 1, and these two numbers are therefore said to be "prime to each other," although neither is a "prime number."

Learn by heart: *Numbers whose only common measure is unity are said to be* PRIME TO EACH OTHER, *even though they be not prime numbers.*

16. Find G.C.M. of 108 and 1440.

First method. Find *all* the factors of the smaller number.

1) (108
2) (54
3) (36
4) (27
6) (18
9) (12

The largest of these which also measures 1440 is of course the G.C.M. Beginning our trial divisions of 1440 with the greatest of these factors, viz. 108 itself, we find that it is not a measure of 1440.

108)1440(13
360
36

Next try 54; this also is not a measure.

54)1440(26
360
36

Now try 36, which is contained exactly 40 times, and is therefore the G.C.M. required.

36)1440(40
..

EXERCISE LXIV.

Find G. C. M. of:

(1) 84 and 96	(5) 120 and 150	(9) 28 and 42
(2) 48 and 144	(6) 38 and 57	(10) 66 and 99
(3) 32 and 60	(7) 28 and 49	(11) 100 and 175
(4) 45 and 28	(8) 141 and 74	(12) 180 and 240

This method, which is so obvious, is convenient for small numbers, but ceases to be so when the factors cannot be readily detected.

Second method. Resolve both numbers into their prime factors.

```
2)108        2)1440
 2)54         2)720
 3)27         2)360
  3)9         2)180      108=2×2×3×3×3
    3          2)90     1440=2×2×2×2×2×3×3×5
               3)45
               3)15
```

From § 12 it is evident that the product of all the prime factors which they have in common is the G. C. M. required. These are $2 \times 2 \times 3 \times 3 = 36$.

EXERCISE LXV.

Find G. C. M. of:

(1) 512 and 240	(5) 112 and 28	(9) 2100 and 2240
(2) 1760 and 990	(6) 77 and 231	(10) 210 and 1008
(3) 5760 and 7000	(7) 840 and 1440	(11) 1485 and 2160
(4) 212 and 504	(8) 360 and 900	(12) 7040 and 7392

Third method. Find G. C. M. of 4063 and 4541. In this case, the above methods would be too laborious, as we cannot determine the factors by mere inspection.

Although as yet the G. C. M. is unknown, the following facts concerning it are known:

(*a*) It cannot possibly exceed the smaller of the given numbers. (§ 3.) Hence,

(*b*) If the smaller is contained in the greater, it is the G. C. M. of the two numbers.

```
4063) 4541 (1
       478
```

On trial, we find the remainder 478.

(*c*) Any common measure of the two numbers must measure the difference of any of their multiples (§ 6), and hence must measure 478, therefore the G. C. M. we are seeking must be *a* common measure of 478 and 4063. Let us, therefore, find a common measure of the numbers 478 and 4063. Reasoning as before, the G. C. M. we are seeking must measure the difference of any of the multiples of 478 and 4063. Take from 4063 the largest possible multiple of 478, i.e. divide 4063 by 478. Remainder 239.

$$\begin{array}{l} 478)\,4063\,(8 \\ \quad\;\; 239 \end{array}$$

Hence the G. C. M. required must measure 239 and 478. As before, divide 478 by 239. We find $478 = 2 \times 239$.

$$\begin{array}{l} 239)\,478\,(2 \\ \quad\;\;\; \cdots \end{array}$$

We have now shewn that *every* measure of 4063 and 4541 must also measure 239; we proceed to prove the converse, viz., that every measure of 239 must also measure 4063 and 4541. Every measure of 239 measures also its multiple 478 (§ 5), and therefore also $8 \times 478 + 239$ (§ 6), i.e. 4063; and therefore also $1 \times 4063 + 478$ (§ 6), i.e. 4541. But the greatest measure of 239 is 239, which is therefore the G. C. M. required. The sum is commonly worked in the following form:

$$\begin{array}{llll} 4063)\,4541\,(1 & & & \\ \quad\;\;\; \underline{4063} & & & \\ & 478)\,4063\,(8 & & \\ & \quad\;\;\; \underline{3824} & & \\ & & 239)\,478\,(2 & \\ & & \quad\;\;\; \underline{478} & \\ & & \quad\;\;\; \cdots & \end{array}$$

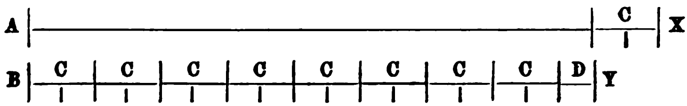
By the method of Ch. XI. § 13, the sum would look thus:

$$\begin{array}{lll} 4063)\,4541\,(1 & & \\ & 478)\,4063\,(8 & \\ & & 239)\,478\,(2 \\ & & \quad\;\;\; \cdots \end{array}$$

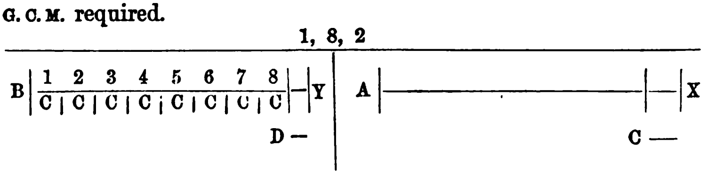
This can be still further contracted by dividing alternately from right to left and left to right, writing the quotients in order at the top of the T.

$$\begin{array}{r|l} \multicolumn{2}{c}{1,\ 8,\ 2} \\ \hline 4063 & 4541 \\ 239 & 478 \\ & \cdots \end{array}$$

The process here followed admits of the following illustration :



It is required to find the greatest length which is contained an exact number of times in each of the straight lines AX and BY. Since this required length (which we may call L) is to measure both AX and BY, it must measure their difference, viz., C ——, and as L measures C and BY, it must measure any number of times C and the difference between BY and this multiple of C; take from BY as many times C as possible, and suppose there is a remainder, D—, after taking it eight times. As before, L must measure both C and D, and suppose we find that D is contained exactly (say twice) in C, D will be the length required; for since D measures C, it measures $8 \times C + D$ or BY; and since it measures BY, it also measures $BY + C$ or AX, and is therefore a common measure of AX and BY; and it has been shewn that L must measure D; and as D is the greatest measure of itself, D is the G. C. M. required.



Exercise LXVI.

Find G. C. M. of:

(1)	2536 and 3487	(11)	78473 and 94653
(2)	2479 and 3589	(12)	2760 and 4485
(3)	3045 and 6195	(13)	1177 and 2675
(4)	8823 and 11937	(14)	14141 and 16289
(5)	568 and 712	(15)	85359 and 86128
(6)	419 and 52301	(16)	44323 and 61087
(7)	11023 and 6493	(17)	17596 and 26145
(8)	27671 and 408870	(18)	1485 and 2160
(9)	35143 and 10283	(19)	7040 and 7392
(10)	232353 and 39699	(20)	999999 and 571428

Fourth method. On examining the process last given, we notice that the question of finding G. C. M. of two large numbers is reduced

to that of finding two smaller numbers having the same G.C.M. If, therefore, one of the two given numbers has a prime factor not contained in the other, it may be thrown out at once. Again, a factor, prime or not, which is common to both, may be taken out and set aside for ultimate multiplication into the last divisor; and all this may be done at any stage of the process. This method will be found useful where the factors can be detected by inspection.

Find G.C.M. of 976800 and 9990.

10)976800	9990	1st stage
3)97680	999	2nd ,,
3256	333	3rd ,,
407	37	4th ,,
37		

∴ G.C.M. $=37\times3\times10=1110$.

1st stage. Take out and preserve the common factor 10.

2nd stage. Reject $5\times2=10$ from the left, neither factor being common; take out and preserve the common factor 3.

3rd stage. Reject $2\times2\times2=8$ from the left, and $3\times3=9$ from the right, as not common.

4th stage. Reject 11 from the left.

Find G.C.M. of 250387 and 41041.

6, 9, 1

250387	41041
41)4141	
101	1001
9	92

G.C.M. $=41$.

In this example, we commenced as usual, but the first division revealed the common factor 41, the two quotients 101 and 1001 proving to be prime to each other. 41 is G.C.M. required.

EXERCISE LXVII.

Find G.C.M. of:

(1) 1679 and 1932	(9) 2268 and 3348
(2) 1003 and 2419	(10) 1189 and 2146
(3) 33853 and 35017	(11) 94653 and 78473
(4) 533 and 1189	(12) 2993 and 3869
(5) 33787 and 34691	(13) 768 and 16777216
(6) 11009 and 12827	(14) 5115 and 7254
(7) 4189 and 4307	(15) 324 and 456
(8) 4489 and 5293	(16) 269178 and 352002

16. If the G. C. M. of more than two numbers be required, we must first find that of any two of them; then the G. C. M. of this result and a third number; then of this second result and a fourth, and so on.

Find G. C. M. of 4994, 7491, 9988, 12485, 16571.

11) 4994	7491
454	681
227	227

G. C. M. 227×11

11) 227×11	9988
227	908
	227

G. C. M. 11×227

11) 11×227	12485
227	1135
	227

G. C. M. 11×227

	73
11×227	16571
227	681
	...

G. C. M. 227

Ans. 227.

227 is therefore the G. C. M. of all the numbers.

Find G. C. M. of 7326, 8547, 9768, 22755.

	7
3) 7326	8547
11) 2442	2849
222	259
111	...
37	

G. C. M. $3 \times 11 \times 37$

	8
3) $3 \times 11 \times 37$	9768
11) 11×37	3256
37	296
	...

G. C. M. $3 \times 11 \times 37$

	41
3) $3 \times 11 \times 37$	22755
11×37	7585
37	1517
	37
	..

G. C. M. $3 \times 37 = 111$

Ans. 111.

NOTE.—If of a series of numbers there be one which measures each of the others, it is the G. C. M. of the series.

EXERCISE LXVIII.

(1) Find G. C. M. of 12, 24, 36.

(2) „ 2255, 4305, 6355, 9020, 10455.

(3) „ 68, 17, 102, 34.

(4) „ 909, 1414, 2323, 4242, 2121.

(5) „ 1521, 585, 4095, 3393, 10764, 4563.

(6) „ 132288, 107328, 138216, 97344.

(7) „ 740, 333, 296.

(8) Find the largest number of which the following are multiples: 833, 1785, 1309.

(9) An exact number of shares, all at the same price, was bought with each of the following sums: £87. 6*s*. 3*d*., £134. 18*s*. 9*d*., £341. 6*s*. 3*d*. Find the highest possible price of each share.

(10) Two distances of 901 and 1037 miles respectively are portioned off into equal daily journeys. Find the smallest number of days in which the journeys can be accomplished.

(11) A court 6 yds., 2 ft., 7 in. long, and 5 yds., 2 ft., 5 in. broad, is to be paved with *square* tiles. Find the largest possible size of the tiles, and how many are required?

17. Least Common Multiple.

The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, &c.; the multiples of 4 are 4, 8, 12, 16, 20, 24, 28, &c. We see that these two numbers have the multiples 12, 24, 36, &c., in common, while 3, 6, 9, 15, 18, &c., belong only to 3, and 4, 8, 16, 20, &c., only to 4.

Learn by heart: *The multiples that two or more numbers have in common are called their common multiples, and the least of these is called their* Least Common Multiple, *which is indicated by the letters* L. C. M.

18. Find L. C. M. of 6 and 7.

Write out simultaneously the multiples of 6 and 7;

6, 12, 18, 24, 30, 36, 42;
7, 14, 21, 28, 35, 42;

until we find a number (42) in one series which has already occurred in the other. This number 42 is the L. C. M. required. Other common multiples will be found further on in the series, but all of them multiples of 42.

Exercise LXIX.

Find L. C. M. of:

(1) 5 and 6	(3) 6 and 11	(5) 9 and 10
(2) 5 and 9	(4) 7 and 3	(6) 2 and 3

The *smallest* (integral) common measure of integers is 1. There cannot be a *greatest* common multiple.

Two numbers must have *a* common multiple; for 2×3 or 6, must be a multiple of 2 and of 3; similarly, 17×19 or 323, must contain 17, 19 times, and 19, 17 times; and the question arises whether this product of two numbers is always not only *a* common multiple, but their *least* common multiple.

Examine the following series, consisting of the several prime numbers and their multiples:

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59
4	6	10	14	22	26	34	38	46	58							
6	9	15	21	33	39	51	57									
8	12	20	28	44	52											
10	15	25	35	55												
12	18	30	42													
14	21	35	49													
16	24	40	56													
18	27	45														
20	30	50														
22	33	55														
24	36	60														
26	39															
28	42															
30	45															
32	48															
34	51															
36	54															
38	57															
40	60															
42																
44																
46																
48																
50																
52																
54																
56																
58																
60																

These series evidently contain every number up to 60; if prime at the head of a column, if composite lower down. Those com-

posite numbers which are powers of prime numbers, will appear only in the column of that prime number, the others will appear in two or more columns; thus 8, which is $2 \times 2 \times 2 = 2^3$, appears only under two, while 6 appears under 2 and 3, and 42 under 2, 3, 7. If the columns were carried far enough in both directions, they could be made to include any number whatsoever. All lists of multiples of composite numbers are dispersed at regular intervals through these series; thus, 6 and its multiples are the 3rd, 6th, 9th, &c., numbers of the (2) column, and the 2nd, 4th, 6th, of the (3) column. 8 and its multiples are only found in the (2) column in the 4th, 8th, 12th, &c., places; 42 and its multiples would be found dispersed with similar regularity in the (2), (3) and (7) columns.

The L.C.M. of 2 and 3 is 6, of 3 and 5 is 15, of 3 and 7 is 21, of 2 and 17 is 34, and so on.

Learn by heart: *The* L.C.M. *of two prime numbers is their product.*

The L.C.M. of 8 and 3 is also their product 24; 8 being only in the (2) and 3 only in the (3) column; the earliest point at which they can coincide is $3 \times 8 = 24$. Similarly, the L.C.M. of 8 and 9 will be found to be 72. The L.C.M. of 6 and 5 or of 2×3 and 5 is 30, for though 6 appears in two columns, 5 does not appear in either of them until we reach 6×5 or 30. Similarly, the L.C.M. of 15 and 4, or of 3×5 and 2×2, is 15×4, or 60. Again, the L.C.M. of 15 and 14, or 3×5 and 2×7, is $15 \times 14 = 210$. We find, then, that if the two given numbers have no factor in common, i.e. if they are *prime to each other*, the L.C.M. is their product. We may therefore extend the last rule.

Learn by heart: *The* L.C.M. *of two numbers prime to each other is their product.**

* This rule is axiomatic in its character, and is accordingly often assumed without proof. We have endeavoured by tabular exhibition to bring it home more closely to the mind of the beginner. It admits, however, of the following proof:

(1) It is proved that if m measures a and b, it measures their G.C.M. (3rd method, pp. 140 and 141.)

(2) If m measures $a \times b$ and is prime to a, it must measure b, for it measures $a \times b$ (hyp) and evidently also $m \times b$, and $\therefore$ the G.C.M. of $a \times b$ and $m \times b$, but

The L.C.M. of 8 and 16 is 16, for it is a multiple of 8, and no less number would be a multiple of 16.

Learn by heart: *If of two numbers one is a multiple of the other, it is the* L.C.M. *of the two.*

Find L.C.M. of 6 and 15. $6 = 2 \times 3$, $15 = 5 \times 3$, $\therefore$ the multiples of each of the given numbers will be found in the (3) column, those of 6 occupying the 2nd, 4th, 6th, 8th, 10th, &c., places, and those of 15 the 5th, 10th, 15th, &c., places. Hence the first place where the multiples coincide is the 10th place, i.e. $10 \times 3 = 30$, because 2 and 5 being prime to one another, 10 is their L.C.M.

Find L.C.M. of 30 and 12. $30 = 2 \times 3 \times 5$, $12 = 2 \times 2 \times 3$. Hence the multiples of 30 and 12 are both in the (2) and in the (3) columns, therefore they would all be in a (6) column, if such were made, or selected from the columns before us. The multiples of 30 would occupy the 5th, 10th, &c., places of such a column, those of 12 the 2nd, 4th, 6th, &c., places; $\therefore$ (as in the case of 6 and 15) $6 \times 5 \times 2$ is L.C.M. of 30 and 12. We have fixed upon the common factor 6 as the largest among whose multiples 30 and 12 could both be found, i.e. the G.C.M. of 30 and 12. Hence to find L.C.M. of two numbers, divide them both by their G.C.M., and the continued product of this G.C.M. and the two quotients will be the L.C.M. required.

Find L.C.M. of 85 and 187.

$$\begin{array}{r|l} 85 & 187 \\ 17 & 17 \end{array} \qquad 17)85(5 \qquad 17)187(11$$
$$\qquad\qquad\qquad\quad \cdot\cdot \qquad\qquad\quad 17$$

$\therefore 5 \times 11 \times 17 = 935$ is L.C.M. required.

One of these divisions might have been dispensed with, since the step of dividing by 17 has to be retraced by subsequent multiplication. We might therefore have proceeded thus:

$$\begin{array}{r|l} 85 & 187 \\ 17 & 17 \end{array} \qquad 17)85(5$$
$$\qquad\qquad\qquad \cdot\cdot$$

$\therefore 5 \times 187 = 935$ is L.C.M. required.

this G.C.M. is b, for if a greater number, say c, were this G.C.M., b would measure c; say $c = k \times b$, $\therefore$ a and m would be divisible by k, and would not be prime to each other as they were supposed to be.

(3) Since m measures b, the least value of b is m, and $\therefore$ $a \times m$ is the L.C.M. of a and m.

Learn by heart: *To find* L. C. M. *of two numbers, divide either number by their* G. C. M., *and multiply this quotient by the other number.*

Find L. C. M. of:

EXERCISE LXX.

(1)	60 and 90	(11)	345 and 346
(2)	75 and 100	(12)	960 and 1000
(3)	80 and 105	(13)	180 and 150
(4)	25 and 75	(14)	801 and 890
(5)	25 and 21	(15)	555 and 370
(6)	13 and 12	(16)	120 and 320
(7)	365 and 657	(17)	424 and 583
(8)	5000 and 6000	(18)	319 and 407
(9)	25 and 30	(19)	1679 and 1932
(10)	345 and 690	(20)	1003 and 2419

19. To find L. C. M. of more than two numbers, we may first find that of any pair, then L. C. M of the result and a third number, and so on; but in most cases this method would be found unnecessarily cumbrous.

20. Find L. C. M. of 8, 24, 48, 96. Since 96 is a multiple of each of the other numbers, and is the least multiple of itself, it is L. C. M. of the series. Hence, if of a series of numbers there is one a multiple of each of the others, this one is the L. C. M. of the series.

21. Examine the number 840. Resolve it into its prime factors.

$$\begin{array}{r|l} 2 & 840 \\ \hline 2 & 420 \\ \hline 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad 840 = 2 \times 2 \times 2 \times 3 \times 5 \times 7$$

If 840 is divided by any of these factors, or by the product of two or more of them, the remaining factors will give the quotient, thus:

$840 \div 2 = 2 \times 2 \times 3 \times 5 \times 7 = 420$. $840 \div 3 \times 7 = 2 \times 2 \times 2 \times 5 = 40$, and so on,

from which we see that 840 is a multiple of any number whose factors are selected from the series 2, 2, 2, 3, 5, 7.

Again, 840 is *not* a multiple of a number whose factors are not *all* to be found in this series.

Find L. C. M. of 30, 56, 42, 140. Resolve each of these numbers into its prime factors.

2)30	2)56	2)42	2)140
3)15	2)28	3)21	2)70
5	2)14	7	5)35
	7		7

$30 = 2 \times 3 \times 5$ $\qquad$ $42 = 2 \times 3 \times 7$

$56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$ $\qquad$ $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

These prime factors can all be selected from the series, 2, 2, 2, 3, 5, 7; therefore, from what we have just said, $2 \times 2 \times 2 \times 3 \times 5 \times 7$, or 840, is *a* common multiple of them all, and it is the *least* common multiple, because no one of these factors can be dispensed with.

Find L. C. M. of 224, 180, 910, 1404, 2025.

2)224	2)180	2)910	2)1404	3)2025
2)112	2)90	5)455	2)702	3)675
2)56	3)45	7)91	3)351	3)225
2)28	3)15	13	3)117	3)75
2)14	5		3)39	5)25
7			13	5

$224 = 2^5 \times 7$ $\qquad$ $1404 = 2^2 \times 3^3 \times 13$

$180 = 2^2 \times 3^2 \times 5$ $\qquad$ $2025 = 3^4 \times 5^2$

$910 = 2 \times 5 \times 7 \times 13$

From these select the highest power of each prime number, viz. 2^5 (from 224), 3^4 (from 2025), 5^2 (from 2025), 7 (from 224 or 910), 13 (from 910 or 1404).

$2^5 \times 3^4 \times 5^2 \times 7 \times 13 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7 \times 13 = 5896800$ is L. C. M. required.

This number 5896800 contains

224,	for it contains	$2 \times 2 \times 2 \times 2 \times 2 \times 7$
180,	,,	$2 \times 2 \times 3 \times 3 \times 5$
910,	,,	$2 \times 5 \times 7 \times 13$
1404,	,,	$2 \times 2 \times 3 \times 3 \times 3 \times 13$
2025,	,,	$3 \times 3 \times 3 \times 3 \times 5 \times 5$

and is therefore a common multiple. Again, no one of the factors can be spared, for if one of the twos were omitted, the result would not contain 224; the absence of one three would vitiate the result as regards 2025, and so on with the other prime factors.

The rule may be stated thus: To find L.C.M. of any numbers, 1st. Resolve each number into prime factors. 2nd. Select from these the highest power of each. 3rd. Find the continued product of these powers. If one of the given numbers is a measure of another of them, it may of course be disregarded. If all the numbers are prime to each other, their continued product is their L.C.M.

EXERCISE LXXI.

Find L. C. M. of:

(1) 2, 3, 5, 7
(2) 2, 3, 6
(3) 3, 5, 9, 25
(4) 3, 5, 15, 9, 25
(5) 6, 60, 12, 15, 20, 30
(6) 4, 8, 12, 16, 20
(7) 7, 14, 15, 21, 45
(8) 30, 40, 50, 60
(9) 16, 25, 81
(10) 80, 200, 45, 72, 225, 48
(11) 98, 35, 77, 121
(12) 26, 39, 52, 65

22. The method of the last paragraph may be shortened.

Find L. C. M. of 80, 200, 40, 45, 72, 225, 48, 36.

1st. Omit 40, 45 and 36, as being measures of 80, 225 and 72, respectively.

2nd. Take out the prime factor 5, common to 80, 200 and 225; divide each of them by 5, giving the series,

$\cancel{16}$, 40, 72, 45, 48.

Omit 16, a measure of 48. Again, taking out 5, we get the series,

8, 72, 9, 48.

Omit 8 and 9, measures of 72, leaving the two numbers 72 and 48. Taking out their G.C.M. 24, we get 2 and 3, prime to each other; hence $5 \times 5 \times 24 \times 3 \times 2 = 3600$ is L.C.M. required.

$$\begin{array}{r|l} 5 & 80,\ 200,\ \cancel{40},\ \cancel{45},\ 72,\ 225,\ 48,\ \cancel{36} \\ 5 & \cancel{16},\ 40,\ \qquad 3,\ 45,\ 2, \\ 24 & \quad \cancel{8}, \qquad \quad \cancel{9}, \end{array}$$

$$\text{L.C.M.} = 5 \times 5 \times 24 \times 3 \times 2 = 3600.$$

This method differs from that in § 21, only in that we take out the common factors from several numbers simultaneously. We may state the rule thus: To find L.C.M. of any numbers, 1st. Expel any of them which measure others. 2nd. Place on the left of the vertical line any prime factor common to two or more of the given numbers, and divide those numbers by it. 3rd. Perform these two

operations on the series thus obtained until we get a series of numbers all prime to one another. 4th. Find the continued product of the last series, and the factors on the left of the vertical line, which will be L. C. M. required.

N.B. We may bring out a composite instead of a prime factor when every member of the series is either divisible by or prime to this composite factor.

EXERCISE LXXII.

Find L. C. M. of:

(1) 24, 20, 18, 16, 12, 15
(2) 18, 36, 24, 35, 20
(3) 6, 10, 14, 15, 21, 35
(4) 30, 42, 105, 70
(5) 12, 20, 28, 18, 30, 42, 45, 63, 105, 70
(6) 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
(7) 21, 15, 33, 35, 77, 105, 165, 385, 231
(8) 16, 6, 8, 2, 12, 3, 48, 24
(9) 34, 26, 65, 85, 51, 39
(10) 10, 20, 30, 40, 50, 60
(11) 105, 120, 616, 88, 24, 12, 6, 308
(12) 12, 18, 27, 63, 28
(13) 7, 11, 4, 14, 10, 5, 15
(14) 323, 247, 209, 133
(15) 6, 5, 3, 11, 35, 44, 68, 17, 14
(16) 1003, 1357
(17) 899, 961
(18) 407, 703, 444
(19) 411, 959, 2055
(20) 120, 400, 500, 375, 1500, 1000, 960

(21) What is the smallest sum of money that can be expressed either as (*a*) guineas or pounds? (*b*) crowns or half-guineas? (*c*) as a multiple of 15*s.* 9*d.* or of 17*s.* 6*d.*?

(22) Find the smallest weight which can be expressed by an exact number either of lbs. troy or lbs. av.

(23) If 1 lb. of sugar is worth $5\frac{1}{2}$*d.* and 1 lb. of coffee 1*s.* 2*d.*; find the smallest number of lbs. of coffee which is worth an exact number of lbs. of sugar.

(24) Find the value of the smallest number of lbs. of coffee at 1*s.* 3*d.* that can be exchanged for an exact number of lbs. of tea at 2*s.* 9*d.* each.

(25) Three travellers journey 15, 18 and 24 miles a-day respectively. How far off is the first station at which all three put up?

(26) If the year of the planet Mercury were exactly 87, that of Venus 225, and of the earth 365 of our days; how many of our years would intervene between any two occasions on which the three planets would appear to a spectator from the sun to be in a straight line?

(27) Of two cog-wheels, with 75 and 120 teeth respectively, a particular tooth of the smaller wheel comes in contact with a tooth of the larger. In how many turns of each wheel will these two teeth meet again, and how many contacts will there have been?

(28) If there be a house-door every 21 yds., and a lamp-post every 44 yds., 1 ft.; supposing a lamp-post exactly opposite one house-door, at what distance will the same occur again, and how many houses and lamp-posts will intervene?

CHAPTER XII.

MISCELLANEOUS EXAMPLES.

1. (1) If 4 articles cost £5. 7*s.* 8*d.*, what will a dozen cost?

(2) If 6 articles cost 17*s.* 3½*d.*, what will 30 cost?

(3) If 20 men can do a piece of work in 17 days, how long will 4 men take?

(4) If 7 silk handkerchiefs cost £1. 8*s.* 10½*d.*, what will 91 cost?

(5) If a dozen of wine cost £2. 2*s.*, what will 3 bottles cost?

(6) If 28 lbs. cost £7. 17*s.* 6*d.*, what will 4 lbs. cost?

(7) Find the cost of 5 articles at £6. 6*s.* 8*d.* per score.

(8) If 17 articles cost £3. 18*s.* 7½*d.*, what will 1 cost?

(9) Find the cost of 15 articles if 8 cost £1. 7*s.* 4*d.* [Here first find the cost of 1 article, and thence of 15.]

(10) Find the cost of 35 articles, if 15 cost £7. 4*s.* 5¼*d.* [15 and 35 not being prime to one another (G. C. M. 5), it is shorter and easier to find the value of 5, and thence of 35.]

(11) If the provisions of a fortress will last 60 days, allowing each man 24 oz. per day, how long will the provisions last, if the allowance is reduced to 15 oz?

(12) And what may be the daily allowance, for the fortress to hold out 144 days?

(13) Find the cost of 9 articles, if 8 cost £17. 4*s.* 10*d.*

(14) Find the cost of 25 articles, if 45 cost £3. 10*s.* 3¾*d.*

(15) If 1 cwt. cost £30, what will 77 lbs. cost?

(16) If 1 ton cost £8. 8*s.* 4*d.*, what will 12 cwt. cost?

(17) If for 20 shillings I can travel 150 miles, how far shall I be able to travel for 12*s.*?

(18) If 2 tons, 5 cwt. cost £23. 12*s.* 6*d.*, what will 1 ton, 5 cwt. cost?

(19) If in 1 year, 8 months, I put by £45, how much shall I put by in $2\frac{1}{3}$ years?

(20) If 1 cwt., 2 qrs., 12 lbs. cost £3. 15*s.* 9*d.*, what will 1 cwt., 3 qrs., 4 lbs. cost?

2. Brackets.

Simplify:

(1) $(8+3)\times 5+10$

(2) $(145+29+10)\times 6+100$

(3) $(2519-728)\times 45+512$

(4) $2519\times 45-728\times 45+512$

(5) $(358+119)\times 7+99$

(6) $99+(119+358)\times 7$

(7) $99+119+358\times 7$

(8) $(99+119+358)\times 7$

(9) $99\times 7+119\times 7+358\times 7$

(10) $67\times 19+25$

(11) $(67+25)\times 19$

(12) $67\times 19+67\times 8$

(13) $67\times(19+8)$

(14) $48520\times 1976+48520\times 1090$

(15) $48520\times(1976+1090)$

(16) $(58512+7426)+(58512-7426)$

(17) $(58512+7426)-(58512-7426)$

[Note that to 16 the answer is twice the greater number, and to 17 twice the less. Why?]

(18) $758\times 758-757\times 757$

(19) $(125796+18043+237509+2759286)\div 602$

(20) $(106033+112568)\div(45+62)$

(21) $(106033+112568)\div(437+598+612+396)$

(22) $(436\times 436-157\times 157)\div(436-157)$

(23) $(8424\times 7056\times 102)\div 9072$

(24) $(3492\times 2049\times 867)\div 15606$

(25) $(34936\times 816\times 2046)\div 209616$

(26) (£1. 12*s.* $8\frac{1}{4}$*d.* $\times 63\times 112$)$\div$£1. 1*s.* $9\frac{1}{2}$*d.*

(27) (£1. 12*s.* $8\frac{1}{4}$*d.* $\times 63\times 112$)$\div 392$

(28) (£597. 13s. 11d. $\times$ 845) — (£597. 13s. 11d. $\times$ 843)

(29) $28519 \times 63 - 28519 \times 53$

(30) $62743 \times 1509 - 62740 \times 1509$

(31) $(14688 \times 1045 \times 10110) \div 108851$

(32) $(812345 + 109876 + 234567 + 321098 + 456789 + 159576) \div (3257 \times 643)$

(33) £14. 7s. $3\frac{3}{4}$d. $\times$ 79 + £14. 7s. $3\frac{3}{4}$d. $\times$ 21

3. (1) London contains 400778 houses, each inhabited by 8 persons on an average. Find the total number of inhabitants.

(2) The water-works of London supply 26 gallons a-day for each person. How much is supplied in a year?

(3) How much will this water weigh, at 10 lbs. per gallon?

(4) In 1868 the births were in London 115744, and the deaths 74908. What was the increase of the population from this source?

(5) The area of the metropolis is 77997 acres. How many square miles is this? (1 square mile = 640 acres.)

(6) The total revenue of the United Kingdom in 1868 was £71,860,677. 12s. 8d., and the expenditure was £74,082,280. 5s. 5d. Find the deficit.

(7) The different sources of inland revenue yielded in 1868: excise, £20,173,288; stamps, £9,461,010; taxes, £3,450,318; income-tax, £6,184,166. The income from the same sources in 1867 was £39,159,781. Find the increase or decrease.

(8) The income-tax was levied at 6d. in the pound. Find the amount of income taxed.

(9) Divide £15,000 among A, B, C and D, giving to A £1961. 0s. 8d. more than to either of the others, who have equal shares.

(10) At an election, the Liberal candidate obtained 1375 votes more than the Conservative. The total number of votes polled was 7209. How many voted for the Conservative?

(11) A leaves an estate of £100,000, of which he disposes as follows: building lodging-houses, £1750; endowment of parish school, 1000 guineas; hospitals, £500; to the church, £500; to each of 4 chapels, £125; to each of 7 old servants, 19 guineas; the residue to be divided amongst his family, giving one-third to his widow, one-fourth to his daughter, and the balance in equal shares to his 2 sons. Find their respective shares.

(12) Find the weekly wages of 325 men, 108 women and 75 children, who receive 4s. 3d., 2s. 10d., 1s. 1d. per day, for each man, woman and child respectively.

(13) A exchanges with B 1 cwt., 11 lbs. of coffee, at $10\frac{1}{2}d.$ per lb., for 5 cwt., 1 qr., 12 lbs. of sugar, at $3\frac{1}{2}d.$ per lb. The difference is to be paid in money. How much is to be paid, and by whom?

(14) Find the average age of 7 persons, aged respectively 47, 38, 52, 45, 41, 49, 43 years. [An average is the sum of a number of quantities divided by the number of these addenda.]

(15) Find the average height of the following Peaks: Monte Viso, 12580 ft.; Genèvre, 11780 ft.; Cenis, 11457 ft.; Izéran, 13266 ft.; M. Blanc, 15732 ft.; Matterhorn, 14835 ft.; Rosa, 15150 ft.; Gallenstock, 12475 ft.; Vogelberg, 10866 ft.; Ortlerspitz, 12852 ft.; Groszglockner, 12776 ft.; Finster-Aarhorn, 14109 ft.; Jungfrau, 13176 ft.

(16) A mixes 4 gallons of spirit at 10s. $9\frac{3}{4}d.$, with 6 gallons at 16s. $1\frac{1}{2}d.$, and 5 gallons at 7s. 6d. each. Find the average cost per gallon.

(17) Find the average age of a school in which there are 20 boys at 9 years old, 4 at 10, 10 at 11, 12 at 12, 11 at 13, 2 at 14, and 1 at 15?

(18) A piece of translation consists of 32 pages, averaging 21 lines of 15 words each. What will be the charge at the rate of 3s. 6d. for every 72 words? Also at the rate of 3s. 6d. for every 96 words?

(19) I mixed 2 cwt., 2 qrs., 20 lbs., at $6\frac{1}{2}d.$ per lb., with 4 cwt., 2 lbs., at 4d. per lb. Find the price per lb. of the mixture.

(20) What is my income, if at 7d. in the £, I pay £11. 1s. 8d. income-tax?

(21) £11. 1s. 8d. $\div$ 7.

(22) A's income is 500 guineas. What will be left him after paying the income-tax of 5d. in the £?

(23) How many times will a coach wheel, of 13 ft., 9 in. in circumference, turn round in going from London to Brighton, 50 miles?

(24) In exchange for 833 articles at 1s. 4d. each, I gave 39 guineas and 100 articles. What was the cost of each of the latter?

(25) Divide £100. 16*s.* among A, B, C and D, giving to A £10. 10*s.* more than to B, and to B £5. 5*s.* more than to either C or D, who have equal shares.

(26) I spent £97. 1*s.* 8*d.* on equal quantities of three kinds of goods, at 4*s.* 5*d.*, 6*s.* 3*d.* and 8*s.* 9*d.* each article respectively. How many articles did I buy?

(27) Divide £171. 10*s.* among 5 men, 6 women and 7 boys, giving to each woman twice a boy's share, and to each man three times a woman's share.

(28) Bought half a ton of sugar for £15, and sold it at $4\frac{1}{4}$*d.* per lb. Find profit or loss.

(29) Find the least number of rupees at 2*s.* 3*d.* each that shall also be an exact number of rupees at 1*s.* $10\frac{1}{2}$*d.* each.

(30) What is the greatest number by which 7927 and 8773 can be divided, leaving remainders 80 and 100 respectively?

(31) Bought 24 yards of cloth for £4. 3*s.* For how much must the whole be sold to gain $6\frac{3}{4}$*d.* per yard?

(32) I sold 243 sheep at £2. 7*s.* 6*d.* each, and with the proceeds bought as many oxen at $16\frac{1}{2}$ guineas each as my money would allow. How many oxen did I buy, and what was over?

(33) How many francs at 10*d.* each can I get for £58. 12*s.* 6*d.*?

(34) Find the cost of 1 ton, 5 cwt., 16 lbs., at $3\frac{3}{4}$*d.* per lb.

(35) A railway, 27 miles in length, is estimated to cost £15000 per mile. How many shares at £25 each must be issued?

(36) Find a number of pounds between £365 and £380, which is also an exact number of guineas.

(37) Find all the sums of money between £280 and £300, which are multiples both of 6*s.* 3*d.* and of 11*s.* 3*d.*

(38) Prove (*a*) that the G. C. M. of any two numbers can never exceed their difference; (*b*) that any two consecutive numbers must be prime to each other; (*c*) that if two numbers are divided by their G. C. M. the quotients are prime to each other; (*d*) that any number which is divisible by two other numbers will be divisible by their L. C. M.; (*e*) that one-third of the difference between any number and the sum of its digits is divisible by 3; (*f*) that every prime

number but 2 can be made composite by the addition or subtraction of unity; (*g*) that every prime number greater than 3 can be made a multiple of 6 either by the addition or else by the subtraction of unity; (*h*) that any two consecutive odd numbers must be prime to each other.

(39) Express 28437 in the undecimal and duodecimal scales.

(40) If I buy 20 gross of pens at 9*d.* per dozen, and sell them at 1*d.* each, what profit do I make?

(41) If I mix 50 gallons of spirit at 14*s.* 3*d.* per gallon, with 64 gallons of water, at what price per gallon must I sell the mixture to gain £8. 1*s.* 6*d.*?

(42) If my salary is 100 guineas per annum, what should I be paid from June 3rd to October 27th?

(43) If my salary is £300 a-year, how much a-year should I lose by being paid £5. 15*s.* per week; and how much should I gain by being paid £5. 15*s.* 6*d.*?

(44) 33 tons of coal, bought at 23*s.* per ton, are sold at 1*s.* 6*d.* per cwt. Find total profit.

(45) Required the weight of 17 boxes, each weighing 2 cwt., 17 lbs.

(46) How many men would weigh a ton, if they weigh on an average 10 stone (of 14 lb.) each?

(47) How many fathoms are there in 17 m., 6 fur., 90 yards?

(48) How many weeks have there been from the beginning of the 19th century to January 6th, 1869, counting leap-years?

(49) If I mix of four different drugs, 5 drs., 2 scr., 14 grs.; 1 oz., 3 drs., 2 scr., 17 grs.; 2 oz., 7 drs., 19 grs., and 6 oz., 4 drs. respectively, and make up the mixture into 26 doses, what will each dose weigh?

(50) If I subscribe 3 guineas the first year, and increase my subscription by 10*s.* 6*d.* each successive year, how much shall I have given in 10 years?

(51) If I begin with £768. 9*s.* 9*d.*, and spend each month one-third of what I have at the beginning of that month, what will be left me after 6 months?

(52) If the Liberal majority in 1868 was 65, and in 1869 was 119, how many seats must have been won?

(53) A sovereign weighs 123 grains; how many can be coined out of 41 oz. troy?

(54) What would 21000 sovereigns weigh in avoirdupois weight?

(55) How long would a velocipede take over 50 miles, at the rate of a furlong a minute?

(56) If a box holds 20 bags of corks, each holding a gross, what will 503 boxes cost at $4\frac{1}{2}d.$ per dozen corks?

(57) The four quarters of the year 1869 begin as follows: March 20th, 1 h., 32 m., p.m.; June 21st, 10 h., 4 m., a.m.; September 23rd, 28 m. a.m.; December 21st, 6 h., 23 m., p.m. Find the lengths of spring, summer and autumn; and taking the year as 365 d., 5 h., 48 m., find the length of winter.

(58) If with $8\frac{1}{2}$ dozen oranges at $1\frac{1}{2}d.$ each, and $31\frac{1}{2}$ lbs. sugar at $5\frac{1}{2}d.$ per lb., I make 45 pots of marmalade, what is the cost per pot?

(59) I spent £291. 5*s.* on equal quantities of 3 different kinds of goods, costing respectively 13*s.* 3*d.*, 18*s.* 9*d.*, £1. 6*s.* 3*d.* each article. How many of each kind did I buy?

(60) How many doses of 5 dwt., 8 grs., can be made of 3 lbs. troy?

(61) A dealer bought 560 sheep at £2. 4*s.* 6*d.* each, and 320 oxen at £18. 10*s.* each. He sold 160 sheep at £2. 18*s.* each, and the remainder at £2. 5*s.* each. Of the oxen, he sold 45 at £20 each, and the remainder at $18\frac{1}{2}$ guineas each. His expenses in the transaction were £37. 10*s.* Did he gain or lose by the transaction, and how much?

(62) Make out a bill for the following purchases:

				s.	*d.*	
12	lbs.	of mutton	@	0	$8\frac{1}{2}$	per lb.
$6\frac{1}{2}$	,,	,,	@	0	9	,,
$14\frac{1}{2}$	,,	beef	@	0	$11\frac{1}{2}$	,,
$8\frac{1}{4}$	,,	,,	@	0	8	,,
$5\frac{3}{4}$	,,	pork	@	1	0	,,
$10\frac{1}{2}$	,,	cheese	@	0	$9\frac{1}{2}$	,,

(63) Find the cost of constructing a railroad 125 miles long, at the rate of 16 guineas per yard.

(64) What do you understand by the prime factors of a number? How may you determine by the inspection of the digits of a number when it is divisible by the numbers 2, 3 and 11 respectively. Find G. C. M. of 7854 and 9768.

(65) Reduce 167805 lbs. avoirdupois to tons.

(66) Find all the divisors of 2145.

(67) I bought 8027 articles at 5*s.* 10¼*d.* each, and sold 4000 articles at 6*s.* 3*d.* each. At how much a-piece must I sell the remainder to make a profit of £309. 17*s.* 7¼*d.* on the whole?

(68) Explain the terms: Measure, prime numbers, odd numbers, and numbers prime to each other.

(69) What sense, if any, can you attach to the following expressions:

a. £15. 3*s.* 8*d.* + £3. 11*s.* 9*d.*
b. £15. 3*s.* 8*d.* — £3. 11*s.* 9*d.*
c. £15. 3*s.* 8*d.* × £3. 11*s.* 9*d.*
d. £15. 3*s.* 8*d.* ÷ £3. 11*s.* 9*d.*
e. £15. 3*s.* 8*d.* × £1.
f. £15. 3*s.* 8*d.* × 0
g. £15. 3*s.* 8*d.* × 1

(70) How many coins, each worth 12*s.* 7*d.*, must be given in exchange for 143 coins at 16*s.* 10½*d.* each, added to 567 coins at 10*s.* 1½*d.* each?

(71) Find all the common measures of 5082, 9438, 10890 and 8712.

(72) If of a series of quotients obtained by dividing each of a given series of dividends by a common measure, one quotient is prime to all the others, the common divisor is the G. C. M. of the series of dividends. Prove it.

(73) If 50 shares are bought at £35. 10*s.* 6*d.* each, and sold for £2042. 18*s.* 4*d.*, what is the gain per share?

(47) A company has 2000 shares at £50 each. It fails, and its debt, £58333. 6*s.* 8*d.* must be paid by the shareholders. How much will the holder of 73 shares have to pay, and what will be his total loss if he bought the shares at half-price?

(75) How long would light, which travels from the sun to the earth in 8 minutes, take from the nearest fixed star, which is 200000 times as far as the sun?

(76) A train started from London at 9.15 a.m., and reached Bristol, 120 miles off, at 12.25 p.m.; it stopped at 5 stations, losing 5 minutes at each, with 15 minutes extra at Swindon for refreshment. Find the average rate of the train.

(77) Divide £6842. 14*s.* 5*d.* among A, B and C, so that A may have £568. 14*s.* 4*d.* more than B, and C £728. 18*s.* 2*d.* less than B.

(78) Find G. C. M. of 15 h. 12 min., and 1 d., 3 h., 33 min.

(79) State and prove the test of accuracy by casting out of nines in division.

[(80) Find the criteria of divisibility by 2, 3, 4 and 5 of numbers expressed in the senary scale.

(81) If the number of odd digits in any number expressed in a scale with an odd radix be even, the number is divisible by 2; if not, not.]

(82) Three watches are set together; the first gains 6, the second 8, and the third 10 minutes a-week. In how many weeks will they again shew the same time?

(83) Convert £5013 to guineas, and 5013 guineas to pounds.

(84) Is 1109 a prime number? Describe the shortest way of deciding the question.

(85) Find G. C. M. of 16776 and 2096, and explain each step of the process. Also of 12018, 20030, and 30045.

(86) Define the Least Common Multiple, and find L. C. M. of 85, 125, 1445, 4913.

(87) Find the cost of 4 tons, 13 cwts., 1 qr., 19 lbs., at $2\frac{3}{4}d.$ per lb., and of 2000 oz. troy, at £3. 17*s.* $10\frac{1}{2}d.$ per oz.

(88) If a certain number of yards at 1*s.* $1\frac{1}{4}d.$ per yard, and the same number of yards at 1*s.* $5\frac{1}{2}d.$ per yard, amount together to £515. 1*s.* 3*d.*, how many yards are there of each?

(89) If 450 articles at £2. 10*s.* 6*d.* per article, and a certain number of articles at 5*s.* $8\frac{1}{2}d.$ each, amount together to £1215. 6*s.* $2\frac{1}{2}d.$, how many articles of the latter kind are there?

(90) If 45 oxen at £18. 12*s.* 6*d.* each, and 75 sheep, together cost £971. 5*s.*, how much does each sheep cost?

(91) A fast train starts $2\frac{1}{2}$ hours after a slow one; in what time will it overtake the former, their rate being 42 and 30 miles per hour respectively?

(92) A and B start from York and London, travelling 23 miles and 17 miles per hour respectively. In what time will they meet, and what distance from London, the total distance being 200 miles?

(93) A house and its furniture together are worth £2085, the value of the house being 4 times that of the furniture. Find the cost of each.

(94) Find the mean of the following observations: $43^0\ 15'$, $48^0\ 43'$, $42^0\ 17'$, $47^0\ 1'$, $50^0\ 50'$, $46^0\ 19'$, 51^0, $44^0\ 10'$, $35^0\ 12'$, $38^0\ 47'$.

(95) Express 4113 (decimal) in the quinary and duodecimal scales.

(96) Convert *tetet* from the duodecimal into the senary scale.

(97) I bought 701 articles for £14493. 3*s.* 6*d.*, and sold them at a profit of £8. 6*s.* 6*d.* each; the total proceeds I invested in some mining shares, each costing £87. 12*s.* 6*d.* and yielding £3. 8*s.* 9*d.* a-year. Find my total yearly income.

(98) The fore wheel of a carriage is 8 feet, 9 inches in circumference; the hind wheel 14 feet, 7 inches. If a nail on the outside of each wheel touch the ground at starting, how many times in the course of a mile will the same two nails be on the ground together?

(99) At the Crystal Palace were admitted on a Foresters' day 83500 persons, each paying 1*s.* How many admissions on a half-crown day would amount to an equal sum?

(100) A person after paying an income-tax of 5*d.* in the £, has £979. 3*s.* 4*d.* left. Find his gross income.

ERRATUM.

P. 42, line 14 from top, for "£632. 11*s.* 2*d.*," read "£630. 11*s.* 2*d.*"

WEIGHTS AND MEASURES.

MEASURES OF LENGTH AND SURFACE.

Lineal Measure.

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
5½ yards......................	= 1 rod or pole (po.)
40 poles, or 220 yards	= 1 furlong (fur.)
8 furlongs, or 1760 yards	= 1 mile (m.)

Gunter's Chain (used for Land-measuring).

100 links, 22 yards, or 4 poles......	= 1 chain (ch.)
484 square yards	= 1 sq. chain
10 sq. chains, or 100,000 sq. links	= 1 acre (ac.)
80 chains	= 1 mile (m.)

Cloth Measure (used by Drapers, Mercers, Clothiers, &c.).

2¼ inches	(in.) ...	= 1 nail (nl.)
9 ,,	(4 nails)	= 1 quarter (qr.)
36 ,,	(4 qrs.)	= 1 yard (yd.)

SQUARE MEASURE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet...............	= 1 square yd (sq. yd.)
30¼ square yards............	= 1 square pole (sq. pl.)
40 square poles	= 1 rood (ro.)
4 roods (4840 yards)......	= 1 acre (ac.)
640 acres	= 1 square mile (m.)

MEASURES OF VOLUME.

Solid or Cubic Measure.

1728 cubic inches	= 1 cubic foot (c. ft.)
27 cubic feet...	= 1 cubic yd. (c. yd.)

MEASURES OF WEIGHT.

Avoirdupois Weight (used in almost all commercial transactions).

16 drams (dr.)...........	=	1 ounce (oz.)
16 ounces..	=	1 pound (lb.) = 7000 grains
28 lbs.....................	=	1 quarter (qr.)
4 quarters, or 112 lbs.	=	1 hundred-weight (cwt).
20 hundred-weight......	=	1 ton (ton)

Troy Weight (used in weighing Gold and Silver, &c.).

24 grains	=	1 pennyweight (dwt.)
20 pennyweights	=	1 ounce (oz.)
12 ounces.........	=	1 pound (lb.) = 5760 grains

MEASURES OF CAPACITY.

Liquid.

4 gills......	=	1 pint (pt.)
2 pints ...	=	1 quart (qt.)
4 quarts...	=	1 gallon (gal.)

Dry Measure.

2 pints (pts.)	=	1 quart (qt.)
4 quarts......	=	1 gallon (gal.)
2 gallons ...	=	1 peck (pk.)
4 pecks	=	1 bushel (bus.)
8 bushels ...	=	1 quarter (qr.)
5 quarters...	=	1 load (ld.)

ANGULAR MEASURE.

60 seconds (″)	=	1 minute (1′)
60 minutes...	=	1 degree (1^0)

MEASURES OF TIME.

60 seconds	=	1 minute (min.)
60 minutes	=	1 hour (hr.)
24 hours	=	1 day (d.)
12 calendar months	=	1 year (yr.)
365 days..............	=	1 year (yr.)

PAPER MEASURE.

24 sheets	=	1 quire (qr.)
20 sheets	=	1 outside quire (out. qr.)
20 quires	=	1 ream (rm.)
10 reams	=	1 bale (bl.)

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